MATH 231 MISSION 6

Same instructions as in Mission 1. See Mission 1 for details.

Problem 76 Your PRINTED NAME below indicates you have:

- (a.) I have read Chapter 6 of Cook: \_\_\_\_\_\_.
- § 15.1 #'s 15, 17, 23, 27, 29, 31, 33, 35, 37, 39, 41, 43, 47, 49
- § 15.2 #'s 3, 5, 7, 9, 13, 15, 17, 21, 23, 25, 27, 29, 31, 39, 45, 49, 51, 55, 65
- § 15.3 #'s 7, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31, 37
- § 15.4 #'s 3, 5, 7, 17, 27
- $\S 15.6 \#$ 's 3, 5, 7, 9, 11, 13, 15, 17, 21, 33, 35, 41, 53
- § 15.7 #'s 1, 3, 5, 7, 9, 11, 17, 19, 21, 23, 29
- $\S 15.8 \#$ 's 1, 3, 5, 7, 9, 11, 13, 17, 19, 21, 23, 25, 27, 35, 41, 43, 48
- § 15.9 #'s 1, 3, 5, 17, 19, 21

**Problem 77** Calculate  $\int_0^1 \int_x^{x^3} y^2 \, dy \, dx$ 

**Problem 78** Calculate  $\int_{0}^{1} \int_{0}^{x} \int_{x^{2}+y^{2}}^{x^{3}+y^{3}} 2z \, dz \, dy \, dx$ 

**Problem 79** Let  $f(x,y) = x^2 + y^3$ . Find the average of f on the square  $[0,1] \times [0,1]$ .

**Problem 80** Reverse the order of integration in order to calculate the following integral:

$$\int_0^1 \int_y^1 \frac{2}{1+x^4} \, dx \, dy.$$

- **Problem 81** Let B be the solid region bounded by x = 0, y = 0, z = 0 and the plane 2x + 2y 3z = 1. Calculate the volume of B.
- **Problem 82** Let R be the regions in quadrant I bounded by  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$  and y = x and y = 3x. Calculate  $\iint_R \sqrt{x^2 + y^2} \, dA$  by changing the integration to polar coordinates.
- **Problem 83** Find the volume bounded by the cylinder  $x^2 + y^2 = 1$  and z = 2 + x + y and z = 1.
- **Problem 84** Let B be a ball of radius R centered at the origin. Calculate  $\iiint_B e^{-\rho^3} dV$ .
- **Problem 85** Find the centroid of the region bounded by  $r = 4\cos\theta$  and r = 2.

- **Problem 86** Suppose the density of a solid sphere B of radius R is given by  $\delta = \alpha \rho^c$  where  $\alpha, c$  are constants and  $\rho = \sqrt{x^2 + y^2 + z^2}$ . The units of the constants are chosen as to give  $\delta = \frac{dm}{dV}$  units of mass per volume. The moment of intertia with respect to the z axis is given by  $I = \iiint_B r^2 \delta dV$  where  $r^2 = x^2 + y^2$ . Calculate I as a function of the constants c and  $\alpha$ .
- **Problem 87** Calculate  $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left(12-x^2-y^2-\sqrt{x^2+y^2}\right) dy dx$  by changing to an integral in polar coordinates.
- **Problem 88** Let P be the parallelogram with vertices (0,0),(0,1),(2,4),(2,5). Calculate

$$\iint_{P} \sqrt{2x(y-2x)} \, dA$$

by making a substitution of x = 2u and y - 2x = v. Notice, this substitution changes P to  $[0,1] \times [0,1]$  in u,v space. Thanks to Brigg's and Cochran section 14.7 for this example.

**Problem 89** Use the Jacobian to relate the volume element  $du d\phi d\theta$  to the Cartesian volume element dx dy dz given that

$$x = u \cos \theta \cosh \phi$$
,  $y = u \sin \theta \cosh \phi$ ,  $z = u \sinh \phi$ .

**Problem 90** Calculate the hypervolume of the set  $\{(x, y, z, t) \mid x^2 + y^2 + z^2 + t^2 \le 1\}$