

Same instructions as in Mission 1. See Mission 1 for details.

**Problem 91** Your PRINTED NAME below indicates you have:

(a.) I have read §7.1 – 7.5 of Cook: \_\_\_\_\_.

(b.) I have attempted 10 problems either from Stewart(see below) or the end of chapter problems in my notes:\_\_\_\_\_.

Same deal as Mission 1. Enjoy:

§ 16.1 #'s 5, 9, 11, 13, 15, 17, 21, 23, 25, 29, 31, 33, 35\*

§ 16.2 #'s 1, 5, 9, 11, 13, 15, 17, 19, 21, 33, 37, 41, 43, 47, 49

§ 16.3 #'s 3, 5, 7, 9, 11, 13, 15, 17, 19, 23, 25, 29, 31, 32, 33, 34, 35

§ 16.5 #'s 1, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17, 19, 21, 22, 25, 27, 28, 31, 33, 37, 38\*

**Problem 92** Let  $f$  be a smooth function and  $\vec{G}$  a smooth vector field on some subset of  $\mathbb{R}^3$ . Show that  $\nabla \cdot (f\vec{G}) = \nabla f \cdot \vec{G} + f\nabla \cdot \vec{G}$  and  $\nabla \times (f\vec{G}) = \nabla f \times \vec{G} + f\nabla \times \vec{G}$

**Problem 93** Let  $f(x, y, z) = x^2 + yz$  and  $\vec{G}(x, y, z) = \langle x, z, z^2 \rangle$ . Let  $\vec{H} = f\vec{G}$  and calculate the curl and divergence of  $\vec{H}$ .

**Problem 94** Let  $k$  be a constant and  $\vec{F}(x, y, z) = \frac{k}{(x^2+y^2+z^2)^{3/2}} \langle x, y, z \rangle$ . Calculate the curl and divergence of  $\vec{F}$ .

**Problem 95** Let  $\vec{B}(x, y, z) = \langle -y, x + z, 2 \rangle$ . Find  $\vec{A}$  for which  $\nabla \times \vec{A} = \vec{B}$ . To my grader: sorry about this. There are infinitely many correct answers.

**Problem 96** Let  $C$  be the curve with parametrization  $x = e^t \cos(t)$ ,  $y = e^t \sin(t)$  and  $z = e^t$  for  $0 \leq t \leq \frac{1}{2} \ln 3$ . Assume the mass-density of  $C$  is uniform and find the center of mass of the wire as well as its arclength.

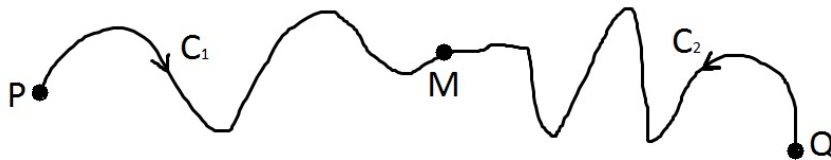
**Problem 97** Let  $C$  be the curve with parametrization  $x = 2 + 3t^2$  and  $y = t^3 - 1$  for  $0 \leq t \leq 1$ . If  $\vec{F}(x, y) = \langle 3x^2 + y, 3y^2 + x \rangle$  then calculate  $\int_C \vec{F} \cdot d\vec{r}$ .

**Problem 98** Let  $\vec{F}(x, y) = \langle x^3 + \sin(y), y^3 + \tan(x) \rangle$ . Calculate the flux of  $\vec{F}$  through the unit-circle  $x^2 + y^2 = 1$ . I recommend you use the divergence form of Green's Theorem for this problem. Direct calculation of the flux seems less fun.

**Problem 99** Let  $R$  be the rectangle with corners  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 3)$  and  $(0, 3)$ . Let  $\partial R$  be the CCW oriented boundary of  $R$  as the notation  $\partial R$  indicates. Calculate by Green's Theorem:

$$\oint_{\partial R} (\cosh(\sqrt{x} + 3) + y) dx + (3 + x^2) dy.$$

**Problem 100** Suppose  $\int_{C_1} \vec{F} \cdot d\vec{r} = 13$  and  $\int_{C_2} \vec{F} \cdot d\vec{r} = -29$ . Let  $C$  be the path from  $P$  to  $M$  to  $Q$  which follows the same set of points as  $C_1$  and  $C_2$ . Calculate  $\int_C \vec{F} \cdot d\vec{r}$ .



**Problem 101** Let  $C$  be a path from  $(0, 1)$  to  $(2, 3)$ . Calculate  $\int_C (1 + 4x^3)dx + (2 - 2y^2)dy$ .

**Problem 102** Find the work done by the force  $\vec{F}(x, y, z) = \langle y + x^2, x, z^3 \rangle$  on a mass as it moves from  $(0, 0, 0)$  to  $(1, 2, 3)$ .

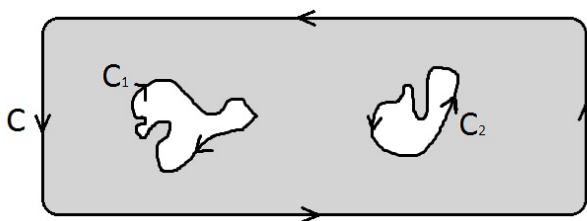
**Problem 103** Determine if the vector fields below are conservative. Find potential functions where possible.

(a.)  $\vec{F}(x, y) = \langle 2x - \sin(x + y^2), -2y \sin(x + y^2) \rangle$

(b.)  $\vec{F}(x, y) = \langle -y, x \rangle$

(c.)  $\vec{F}(y, z) = \langle z + y^2, y + z^3 \rangle$

**Problem 104** Suppose we are given  $\int_{C_1} \vec{F} \cdot d\vec{r} = -10$  and  $\int_{C_2} \vec{F} \cdot d\vec{r} = 9$ . Calculate  $\int_C \vec{F} \cdot d\vec{r}$  given that  $\vec{F}$  is conservative (locally) on the domain between  $C$  and the curves  $C_1, C_2$ . Is  $\vec{F}$  conservative on  $\mathbb{R}^2$ ? Explain.



**Problem 105** Let  $\vec{F} = \langle 0, 0, -mg \rangle$  where  $m, g$  are positive constants and suppose  $\vec{F}_f = -b\vec{T}$  where  $v$  is your speed and  $b$  is a constant and  $\vec{T}$  is the unit-vector which points along the tangential direction of the path. This is a simple model of the force of kinetic friction, it just acts opposite your motion. Find the work done by  $\vec{F}_f + \vec{F}$  as you travel up the helix  $\vec{r}(t) = \langle R \cos(t), R \sin(t), t \rangle$  for  $0 \leq t \leq 4\pi$ .