MATH 231 MISSION 8

Same instructions as in Mission 1. See Mission 1 for details.

Problem 106 Your Printed Name below indicates you have:

- (a.) I have read $\S 7.6 7.10$ of Cook:
- (b.) I have attempted 10 problems either from Stewart(see below) or the end of chapter problems in my notes:_______.
- § 16.4 #'s 1, 3, 5, 7, 9, 11, 13, 17, 19, 21*, 22, 23, 29
- § 16.6 #'s 1, 3, 5, 13, 15, 19, 21, 23, 25, 29, 33, 35, 39, 41, 45, 49
- § 16.7 #'s 5, 7, 9, 11, 13, 19, 21, 23, 25, 27, 29, 31, 39, 45, 49
- § 16.8 #'s 1, 3, 5, 7, 9, 13, 15, 17
- § 16.9 #'s 1, 3, 5, 7, 9, 11, 13, 17

Problem 107 Find the surface area of torus with radii A, R > 0 and $R \ge A$ parametrized by

$$\vec{X}(\alpha, \beta) = \left\langle \left[R + A\cos(\alpha) \right] \cos(\beta), \left[R + A\cos(\alpha) \right] \sin(\beta), A\sin(\alpha) \right\rangle$$

for $0 < \alpha < 2\pi$ and $0 < \beta < 2\pi$.

- **Problem 108** Let S be the outward oriented unit-sphere. Calculate $\iint_S \langle x^3, y^3, z^3 \rangle \cdot d\vec{S}$.
- **Problem 109** Consider a thin-shell of constant density δ . Let the shell be cut from the cone $x^2 + y^2 z^2 = 0$ by the planes z = 1 and z = 2. Find (a.) the center of mass and (b.) the moment of intertia with respect to the z-axis.
- **Problem 110** Find the flux of $\vec{F}(x,y,z) = \langle z^2, x, -3z \rangle$ through the parabolic cylinder $z = 4 y^2$ bounded by the planes x = 0, x = 1 and z = 0. Assume the orientation of the surface is outward, away from the x-axis.
- **Problem 111** Find the flux of $\vec{F}(x,y,z) = \langle -x, -y, z^2 \rangle$ through the conical frustrum $z = \sqrt{x^2 + y^2}$ between the planes z = 1 and z = 2 with outward orientation.
- **Problem 112** Suppose \vec{C} is a constant vector. Let $\vec{F}(x,y,z) = \vec{C}$ find the flux of \vec{F} through a surface S on plane with nonzero vectors \vec{A}, \vec{B} . In particular, the surface S is parametrized by $\vec{r}(u,v) = \vec{r}_o + u\vec{A} + v\vec{B}$ for $(u,v) \in \Omega$.
- **Problem 113** Let $\phi = \pi/4$ define a closed surface S with $0 \le \rho \le 2$. Find the flux of

$$\vec{F}(\rho, \phi, \theta) = \phi^2 \hat{\rho} + \rho \hat{\phi} + \hat{\theta}$$

through the outward oriented S.

Problem 114 Consider the closed cylinder $x^2 + y^2 = R^2$ for $0 \le z \le L$. Find the flux of

$$\vec{F}(r,\theta,z) = \theta \, \hat{z} + z \hat{\theta} + r^2 \hat{r}$$

out of the cylinder.

- **Problem 115** Let $\vec{F}(x,y,z) = \langle y^2 + z^2, x^2 + z^2, x^2 + y^2 \rangle$. Find the work done by \vec{F} around the CCW (as viewed from above) triangle formed from the intersection of the plane x + y + z = 1 and the coordinate planes. (use Stoke's Theorem)
- **Problem 116** Let $\vec{F} = \langle 2x, 2y, 2z \rangle$ and suppose S is a simply connected surface with boundary ∂S a simple closed curve. Give two arguments (one by Stokes', the other by Gauss' theorem) that $\int_{\partial S} \vec{F} \cdot d\vec{r} = 0$.
- **Problem 117** Suppose S is the union of the cylinder $x^2 + y^2 = 1$ for $0 \le z \le 1$ and the disk $x^2 + y^2 \le 1$ at z = 1. Suppose \vec{F} is a vector field such that

$$\nabla \times \vec{F} = \left\langle \sinh(z)(x^2 + y^2), ze^{xy + \cos(x+y)}, (xz + y) \tan^{-1}(z) \right\rangle.$$

Calculate the flux of $\nabla \times \vec{F}$ though S.

Problem 118 Let E be the cube $[-1,1]^3$. Calculate the flux through ∂E of the vector field

$$\vec{F}(x, y, z) = \langle y - x, z - y, y - x \rangle$$

(please use the divergence theorem!)

- **Problem 119** Calculate the flux of $\vec{F}(x, y, z) = \langle 1, 2, z^2 \rangle$ out of the paraboloid $z = 4 x^2 y^2$ bounded below by the xy-plane.
- **Problem 120** Let a spherical shell S of radius R and total mass M have constant mass density $\sigma = \frac{dm}{dS}$. Find the moment of intertia for this shell with respect to the z-axis. In particular, calculate

$$\iint_{S} \sigma r^2 dS.$$