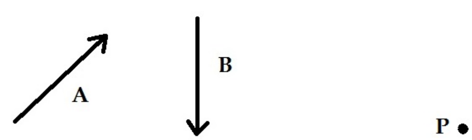
Please work the problems in the white space provided and clearly box your solutions. You are allowed one $3'' \times 5''$ notecard. Enjoy!

Problem 1 (5pts) Find a nonzero vector which is orthogonal to both $\widehat{x}$ and $\widehat{y} + \widehat{z}$.

Problem 2 (9pts) Find the magnitude, standard-angle and unit-vector for $\vec{B} = \langle -2, \sqrt{3} \rangle$.

Problem 3 (6pts) For the vectors pictured below. Draw $\vec{A} - \vec{B}$ and explain which direction $\vec{A} \times \vec{B}$ points. Please draw $\vec{A} - \vec{B}$ starting at the point P.



Problem 4 (15pts) Let $\vec{A} = \langle 1, 1, 1 \rangle$ and $\vec{B} = \langle 0, 3, 4 \rangle$. Find the following:

(a) a vector of length 4 in the direction of $\vec{B}$

(b) the projection of $\vec{A}$ in the direction of $\vec{B}$

(c) the angle between $\vec{A}$ and $\vec{B}$

Problem 5 (7pts) A plane contains the point $(1, 1, 1)$. Also, the line $\vec{r}(t) = \langle 2 + t,\, 3t,\, 6\rangle$ is normal to the plane. Find the Cartesian equation of the plane.

Problem 6 (10pts) Find the Cartesian equation of the plane containing the points $P = (1, 1, 1)$, $Q = (0, 1, 0)$ and $R = (0, 0, 1)$.

Problem 7 (5pts) Suppose S is a surface is the collection of all $(x, y, z) \in \mathbb{R}^3$ such that $z^2 = x^2 + y^2$. Find a parametrization of S which does not use square roots and sketch and name the surface.

Problem 8 (14pts) Let $\vec{\gamma}(t) = \langle 5\sin t, 4\cos t, 3\cos t\rangle$ for $t \geq 0$. Find the arclength function of this curve based at $t = 0$. Also, find the T, N and B vectorfields at time t (or arclength s if you prefer). Finally, find the curvature and torsion of the curve.

Problem 9 (10pts) Suppose $\vec{a} = t\,\widehat{x}$ is the acceleration at time t for a ninja. Given that the ninja is initially springing into action with velocity $\langle 1, 2, 3\rangle$ at the position $(0, 0, 0)$ find both the velocity and position of the ninja at time t.

Problem 10 (5pts) Show $\dfrac{d}{dt}\big[\vec{A} \bullet \vec{B}\big] = \dfrac{d\vec{A}}{dt} \bullet \vec{B} + \vec{A} \bullet \dfrac{d\vec{B}}{dt}$. You may assume $\vec{A}, \vec{B}$ are vectors in two-dimensions.

Problem 11 (5pts) Calculate, and simplify as much as possible, the following derivative:

$$\frac{d}{dt}\big[\vec{r} \bullet (\vec{r}\,' \times \vec{r}\,'')\big]$$

Problem 12 (4pts) Parametrize the ellipse $x^2/a^2 + y^2/b^2 = 1$ found in the $z = 3$ plane.

Problem 13 (4pts) Find the equation of the sphere $x^2 + y^2 + z^2 = R^2$ in cylindrical coordinates.

Problem 14 (6pts) Suppose $\langle 4, 7\rangle = s\vec{A} + t\vec{B}$ where $\vec{A} = \langle 1, 1\rangle$ and $\vec{B} = \langle 1, -1\rangle$. Find s and t.

Problem 15 (5pts) Let $\vec{A}, \vec{C}$ be nonzero, non-colinear vectors. Let γ be a curve parametrized by:

$$\vec{\gamma}(t) = \vec{r_o} + f(t)\vec{A} + g(t)\vec{C}$$

for $t \in \mathbb{R}$ where $f, g : \mathbb{R} \to \mathbb{R}$ are smooth functions and $\vec{r_o}$ is a constant vector. Find the torsion of γ.