Math 321: Jordan and Real Jordan Form	Quiz 4 [20pts]

Your printed name indicates you read Chapter 5 of the notes: ____________________.

Problem 1 Friedberg, Insel and Spence 5th edition, §5.4#2a, c, e, page 320.

Problem 2 Friedberg, Insel and Spence 5th edition, §5.4#6, page 320

Problem 3 Friedberg, Insel and Spence 5th edition, §5.4#36, page 325

Problem 4 Friedberg, Insel and Spence 5th edition, §7.2#4a, b, page 503

Problem 5 Friedberg, Insel and Spence 5th edition, §7.2#5a, d, page 504

Problem 6 Friedberg, Insel and Spence 5th edition, §7.3#3, page 515

Problem 7 Friedberg, Insel and Spence 5th edition, §7.3#8, page 516

Problem 8 Let $A = \begin{bmatrix} -6 & -4 \\ 10 & 6 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A. Is A diagonalizable as a real matrix? Is A diagonalizable as a complex matrix? Find the real Jordan form associated with A.

Problem 9 Let $A = \begin{bmatrix} 2 & 4 & -4 \\ -1 & 2 & -1 \\ 1 & 4 & -3 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A. Is A diagonalizable as a real matrix? Is A diagonalizable as a complex matrix? Find the real Jordan form associated with A.

Problem 10 Let $A = \begin{bmatrix} -5 & 4 & 0 & -4 \\ -11 & 8 & -2 & -7 \\ 2 & -1 & 1 & 3 \\ -1 & 1 & -2 & 0 \end{bmatrix}$. You can check this matrix has eigenvalues of $\lambda = 1 \pm 2i$ repeated. In fact (and please, understand, I do **not** want you to actually find these vectors) there exist nonzero vectors $v_1 = a_1 + ib_1$ and $v_2 = a_2 + ib_2$ such that:

$$(A - (1+2i)I)v_1 = 0 \qquad \& \qquad (A - (1+2i)I)v_2 = v_1$$

If $L(x) = Ax$ then find $[L]_{\beta,\beta}$ and $[L]_{\gamma,\gamma}$ with respect to the bases $\beta = \{a_1, b_1, a_2, b_2\}$ and $\gamma = \{v_1, v_2, \overline{v_1}, \overline{v_2}\}$.

Problem 11 Suppose V is a vector space of dimension 4 over $\mathbb{R}$ and $T : V \to V$ is a linear transformation and there exist nonzero vectors v_1, v_2, v_3, v_4 such that:

$$T(v_1) = 7v_1 + v_2, \qquad T(v_2) = 7v_2, \qquad (T - 4Id_V)(v_3) = 0, \qquad T(v_4) = 4v_4$$

Add a needed condition (if any) and find a Jordan basis β for T and calculate $[T]_{\beta,\beta}$. Also, calculate $\det(T)$ and $\text{trace}(T)$.

Problem 12 Suppose A is a 6×6 real matrix with characteristic polynomial $p(t) = (t-3)^3(t-2)^2(t-1)$. What are the possible Jordan forms associated to A. For each form determine the minimal polynomial for A.

Problem 13 Let $T : V \to V$ have basis $\beta = \{v_1, \dots, v_n\}$ for which the matrix of T is in Jordan form:

$$[T]_{\beta,\beta} = J_4(3) \oplus J_2(3) \oplus J_1(3) \oplus J_1(3) \oplus J_4(6)$$

Select vectors from β to construct the basis for each eigenspace and generalized eigenspace for T. That is, find $\beta_j \subset \beta$ for which $\mathcal{E}_{\lambda_j} = \text{span}(\beta_j)$ and $\gamma_j \subset \beta$ for which $\text{span}(\gamma_j) = K_\lambda$ for each eigenvalue of T.

Problem 14 Show $P^{-1}e^{tA}P = e^{tP^{-1}AP}$ for any invertible matrix $P \in \mathbb{C}^{n\times n}$ and matrix $A \in \mathbb{C}^{n\times n}$.

Problem 15 If $A = J_2(3) \oplus J_3(7)$ then calculate e^{tA}.

Problem 16 Friedberg, Insel and Spence 5th edition, §5.2#15b, c, page 280.