Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 1 Your signature below indicates you have:

(a.) I read Chapter 1 and 2 of Cook's lecture notes: ____________________.

Problem 2 Let $S = \{a, b, c\}$ where a, b, c are **distinct** elements of S.

(a.) if we view S as a **set** then $S \cup \{a, b\}$ is:

(b.) if we view S as a **multiset** then $S \cup \{a, b\}$ is:

Problem 3 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$ for all $x \in \mathbb{R}$. Let $a \in \mathbb{R}$. Find the inverse image of the singleton $\{a\}$. Break into cases as needed.

Problem 4 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x-1}{x-3}$ for all $x \in \mathbb{R} - \{3\}$. Also, define $f(3) = 1$. Show that f is a 1-1 and onto function. In other words, show that f is both injective and surjective or, simply, show f is a bijection.

Problem 5 Let $a_i, b_i \in \mathbb{C}$ for all $i \in \mathbb{N}$. Prove by induction that $\sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i = \sum_{i=1}^{n}(a_i + b_i)$. Your proof should include explicit comment about the use of definition of the finite sum as well as the properties of complex arithmetic. (we can replace $\mathbb{C}$ with any other set which allows addition just the same, I chose $\mathbb{C}$ to be concrete.)

Problem 6 A matrix is defined by $A_{ij} = i + j^2$ for $1 \leq i \leq 2$ and $1 \leq j \leq 3$. Write A explicitly.

Problem 7 Write the following system of equations in matrix-column notation $Av = b$ (indicate A,v and b explicitly)

$$x + y + z = 18, \qquad 2x - z = -3, \qquad x + y - z = 0.$$

Problem 8 For the previous problem, write the system as an augmented coefficient matrix. Then find solution **set** via row-reduction. Explicitly denote each row-operation with the short-hand arrow notation as shown in lecture.

Problem 9 Write the following system of equations in matrix-column notation $Av = b$ (indicate A,v and b explicitly)

$$x_1 + x_2 + x_3 + x_4 = 10, \qquad 2x_1 + 2x_2 + 4x_4 = 4$$

Problem 10 For the previous problem, write the system as an augmented coefficient matrix. Then find the standard form of the solution **set** via row-reduction. Explicitly denote each row-operation with the short-hand arrow notation as shown in lecture.

Problem 11 Write the following system of equations in matrix-column notation $Av = b$ (indicate A,v and b explicitly)

$$x + y + z = 3, \qquad 2x + y + z = 5, \qquad 3x - y + z = 0, \qquad 2x + 2y + 2z = 7.$$

Problem 12 For the previous problem, write the system as an augmented coefficient matrix. Then find the standard form of the solution **set** via row-reduction. Explicitly denote each row-operation with the short-hand arrow notation as shown in lecture.

Problem 13 Indicate with $*$-notation for unknown, possibly nonzero numbers, all the possible formats for the **reduced row echelon form** of a 2×4 matrix. For example, for a 1×2 matrix A the possibile formats of rref(A) are $[0\ 0]$, $[1\ *]$ and $[0\ 1]$.

Problem 14 Find all solutions of the matrix equation $\begin{bmatrix} x^2 & y \\ y^2 & x \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 9 & 2 \end{bmatrix}$.

Problem 15 Let E_{ij} be 2×2 matrices defined by $(E_{ij})_{kl} = \delta_{ik}\delta_{jl}$ for $i, j, k, l \in \mathbb{N}_2 = \{1, 2\}$. Write the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ as linear combination of the standard unit-matrices E_{ij}.

Problem 16 Find all quadratic polynomials whose graphs contain $(1, 1), (2, 1), (3, 1)$.

Problem 17 Find all polynomials whose graphs contain $(1, 1), (2, 1), (3, 1)$.

Problem 18 Solve the system

$$\sin\alpha + \cos\beta + 2\sin\gamma = \sqrt{2} + 1, \qquad 2\sin\alpha + 3\cos\beta - \sin\gamma = 1 - \sqrt{2}, \qquad 3\sin\alpha + \cos\beta = 3$$

by making a change of variables. How many solutions do you obtain? Why does this not contradict the general comment we made about the number of solutions to a linear system?

Problem 19 Consider $\mathbb{Z}/2\mathbb{Z} = \{\bar{0}, \bar{1}\}$. List all the distinct 2×2 matrices over $\mathbb{Z}/2\mathbb{Z}$.

Problem 20 Consider $\mathbb{Z}/6\mathbb{Z} = \{\bar{0}, \bar{1}, \bar{2}, \dots \bar{5}\}$. Scalar multiply $\begin{bmatrix} \bar{0} & \bar{1} & \bar{2} \\ \bar{3} & \bar{4} & \bar{5} \end{bmatrix}$ by $\bar{3}$. What is strange about the products in the $(1, 3)$ and $(2, 2)$ components?