Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

**Note:** In this assignment, $R$ denotes a commutative ring.

**Problem 1** Your signature below indicates you have:

**(a.)** I read up to §2.5 of Cook's lecture notes: ____________________.

**(b.)** I read Chapter 1 of Curtis' text: ____________________.

**Problem 2** Let $S = \{a, b, c\}$ where $a, b, c$ are **distinct** elements of $S$.

**(a.)** if we view $S$ as a **set** then $S \cup \{a, b\}$ is:

**(b.)** if we view $S$ as a **multiset** then $S \cup \{a, b\}$ is:

**Problem 3** Let $z = 3 + i$ and $w = -3 + 4i$. Put the following into **Cartesian form**

(a) $\dfrac{1}{z} + \dfrac{1}{w}$

(b) $\dfrac{z^2 + 3wz - 4w^2}{z^2 - w^2}$

**Problem 4** Following the example in my notes where I show the multiplication and addition table for $\mathbb{Z}/7\mathbb{Z}$, create the addition and multplication tables for $\mathbb{Z}/5\mathbb{Z}$. Use your table to find the multiplicative inverse for each nonzero element in $\mathbb{Z}/5\mathbb{Z}$. Given that $\mathbb{Z}/5\mathbb{Z}$ is a ring, are your observations sufficient to prove $\mathbb{Z}/5\mathbb{Z}$ is a field ?

**Problem 5** Following the example in my notes where I show the multiplication and addition table for $\mathbb{Z}/7\mathbb{Z}$, create the addition and multplication tables for $\mathbb{Z}/8\mathbb{Z}$. Use your table to find multiplicative inverses for each unit in $\mathbb{Z}/8\mathbb{Z}$. Also, list all zero divisors in $\mathbb{Z}/8\mathbb{Z}$. Explain why $\mathbb{Z}/8\mathbb{Z}$ is **not** a field.

**Problem 6** Show that if $a \in R$ is a zero divisor then $a$ cannot have a multiplicative inverse.

**Problem 7** Complete Problem 1(a) on page 14 of Curtis' text (basic induction problem).

**Problem 8** Let $c, A_i \in R$ for all $i \in \mathbb{N}$. Use induction to prove that $\sum_{i=1}^{n}(cA_i) = c\sum_{i=1}^{n} A_i$.

**Problem 9** Complete Problem 3 on page 15 of Curtis' text (binomial theorem).

**Problem 10** Prove (3.) of Prop. 2.2.6 in my notes; that is, show $c_1(A + B) = c_1A + c_1B$ for $c_1 \in R$ and $A, B \in R^{m\times n}$.

**Problem 11** Prove (1.) of Theorem 2.3.11 in my notes; that is, show $(AX)Z = A(XZ)$ for multipliable matrices $A, X, Z$ over $R$.

**Problem 12** Let $A \in R^{m \times n}$ and $B \in R^{n \times p}$. Prove that $(AB)^T = B^T A^T$.

**Problem 13** The socks-shoes identity is easily generalized. Let $A_i \in R^{n \times n}$ for each $i \in \mathbb{N}$. Use induction to prove $(A_1 A_2 \cdots A_n)^T = A_n^T \cdots A_2^T A_1^T$ for all $n \in \mathbb{N}$

**Problem 14** Let us define the set of component-wise nonzero $2 \times 2$ matrices over $\mathbb{Z}/3\mathbb{Z}$ by

$$M = \{A \in (\mathbb{Z}/3\mathbb{Z})^{2\times 2} \mid A_{ij} \neq 0 \text{ for all } (i,j) \in \mathbb{N}_2 \times \mathbb{N}_2\}$$

List all the matrices in $M$.

**Problem 15** Find which of the matrices in $M$ of the previous problem are invertible. Find the inverse of each invertible matrix in $M$ and express them in the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \in \{\overline{0}, \overline{1}, \overline{2}\}$. Notice, the formula in Example 2.4.6 works for any field $\mathbb{F}$ and $\mathbb{Z}/3\mathbb{Z}$ is a field.

**Problem 16** Let $A = \begin{bmatrix} \overline{0} & \overline{1} \\ \overline{6} & \overline{4} \end{bmatrix}$ over $\mathbb{Z}/8\mathbb{Z}$. Calculate the following:

(a) $A + A$
(b) $A + 7A$
(c) $A^2$
(d) $A^4$

**Problem 17** If $v = [1, 2, 3]$ and $B = \begin{bmatrix} 4 & 5 \\ 1 & 1 \\ 0 & 6 \end{bmatrix}$ then calculate the following if possible (or explain why it is not possible)

(a) $vB$
(b) $Bv$
(c) $vBv^T$
(d) $BB^T$
(e) $B^T B$

**Problem 18** If $e_i^T A e_j = i^2 + j^2$ for all $(i,j) \in \mathbb{N}_3 \times \mathbb{N}_2$ then find $A$ explicitly as an array of numbers.