Please follow the format which was announced in Blackboard. Thanks!
Your PRINTED NAME indicates you have read through Chapters 1 and 2 of my notes: ____________________.

Problem 1 Let $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$. Calculate the following if possible (otherwise, explain why it can't be done)

(a.) $A + A^2$

(b.) $B + B^2$

(c.) $A + BB^T$

(d.) $A + B^T B$

Problem 2 Let $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$. Prove $A^n = \begin{bmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{bmatrix}$ for all $n \in \mathbb{N}$.

Problem 3 A standard notation is that $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ in context. We could write $I = I_2$ to emphasize the size if needed. Calculate $I(a, b)$ and $[a, b]I$.

Problem 4 Let $M = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and $N = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$ where $a, b, x, y \in \mathbb{R}$.

(a.) Calculate MN. Relate your result to the product of the complex numbers $(a + ib)(x + iy)$.

(b.) Calculate M^{-1} given that at least one of a or b is nonzero. Relate your result to cartesian form of $1/(a + ib)$.

Problem 5 Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$. Choose appropriate sized standard basis and **square** matrix units for the following calculations to make sense and calculate:

(a.) Ae_2

(b.) $e_1^T A$

(c.) AE_{12}

(d.) $E_{12}A$

Problem 6 Suppose A is a symmetric matrix and B is nilpotent of order 3. Let $M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$. Is M symmetric? Is M^2 symmetric? Is M^3 symmetric. Comment on M^k, for which k can we be certain M^k a symmetric matrix?

Problem 7 Let A, B, N, S be invertible square matrices. Solve the following equation for X:

$$(BA)^{-1} X S^{-1} = (NA)^2$$

Problem 8 Prove Proposition 2.2.6 part (2.) in my notes.

Problem 9 Prove the column-by-column multiplication rule for $A \in R^{m\times n}$ and $B \in R^{n\times p}$. That is, if $B = [B_1|B_2|\cdots|B_p]$ then show

$$AB = [AB_1|AB_2|\cdots|AB_p].$$

Problem 10 Let A, B be matrices over R which can be multiplied. Prove $(AB)^T = B^T A^T$.

Problem 11 For each system below, find a matrix A and a vector of variables v such that the systems below are equivalent to $Av = b$. Also, state the augmented coefficient matrix for each system.

a.) $x + y = 2, \quad x - y = 1.$

b.) $a - b = 1 + c, \quad b = 3 - c.$

c.) $x_1 + x_2 = 1, \quad x_2 + x_3 = 0, \quad x_3 + x_4 = 1, \quad x_4 + x_5 = 0.$

Problem 12 Solve each system in the previous problem and write the solution **set** with proper **set** notation. Feel free to use any reasonable method to find the solution. Assume these are equations over $\mathbb{Q}$.

Problem 13 Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Define $X = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$. Order variables in the order x, y, z, w and find the augmented coefficient matrix for the linear system $AX + XB = I$.

I realize some of you have not taken Math 231, but, that is no reason to avoid questions of geometry! If anything, that is a reason we should do more of it in here. Towards that end, here are two geometry problems which I'd like you to solve with algebra. Do not take a geometric approach to the following two problems. Thanks!

Problem 14 The equation of a plane has the form $Ax + By + Cz = D$ when the normal to the plane is $\langle A, B, C\rangle$. Suppose the points $(1, 2, 2), (2, 3, 3)$ and $(0, 0, 2)$ are on a given plane. Find the possible normals for the given plane.

Problem 15 Consider the points $(1, 2), (3, 4), (8, 6)$. Show these points are not on a line. We define a **line** in the plane to be the solution set of $Ax + By = C$ where A, B, C are constants where at least one of A, B is nonzero.

Problem 16 Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 3 & 1 & 1 & 0 \\ 0 & 4 & 4 & 0 & 0 \end{bmatrix}$. Use row reduction to calculate rref(A).

Problem 17 Let $B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 2 \\ 3 & 5 & 0 \end{bmatrix}$. Use row reduction to calculate rref(B).

Problem 18 Let $M = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & -1 \end{bmatrix}$. Calculate rref$(M)$ and rref(M^T) via row reduction.