Your PRINTED NAME indicates you read Chapter 1 and §2.1, 2.2 of the notes: ____________________.

Assume R is a commutative ring with identity throughout this homework. Also, $\mathbb{N} = \{1, 2, \dots\}$.

Problem 1 Consider the system of equations in $\mathbb{Z}_7$:

$$\begin{aligned} x + 2y &= 1 \\ -2x + 7y &= 2 \end{aligned}.$$

Find the solution set by writing this system as $Av = b$ and then multiplying by A^{-1} to obtain $v = A^{-1}b$. *Hint: you ought to be able to calculate A^{-1} as we did in lecture*

Problem 2 Consider the system of equations in $\mathbb{Z}_{17}$:

$$\begin{aligned} x + 5y &= a \\ -2x + 7y &= b \end{aligned}.$$

Find the solution set through row-reduction or another technique. In the following cases:

(a.) $a = 1$, $b = 2$

(b.) $a = b = 0$.

Problem 3 Suppose $a_i, b_i, c \in R$ for $i \in \mathbb{N}$. Prove $\displaystyle\sum_{i=1}^{n}(ca_i + b_i) = c\sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$ for all $n \in \mathbb{N}$.

Problem 4 Prove Theorem 1.3.11.1; prove matrix multiplication is associative for matrices over R.

Problem 5 Let A, B be matrices over R of the same size and let $c \in R$. Prove $(cA+B)^T = cA^T + B^T$.

Problem 6 The **trace** is defined by $tr(A) = \displaystyle\sum_{i=1}^{n} A_{ii}$ for $A \in R^{n\times n}$. Prove that:

(a.) $tr(cA + B) = ctr(A) + tr(B)$ for $c \in R$ and $A, B \in R^{n\times n}$

(b.) $tr(AB) = tr(BA)$ for $A \in R^{m\times n}$ and $B \in R^{n\times m}$.

Problem 7 A diagonal matrix $D = diag(a_1, \dots, a_n)$ can be written as $D = \displaystyle\sum_{i=1}^{n} a_i E_{ii}$. Give a proof my mathematical induction that $D^k = \displaystyle\sum_{i=1}^{n} a_i^k E_{ii}$ for all $k \in \mathbb{N}$.
Hint: I would like for you to use the formulas developed for E_{ij} in Chapter 1 to aid in this proof. Please do not just quote Proposition 1.4.20, however, perhaps the proof I give there will be helpful

Problem 8 Find all $n \times n$ square matrices over R which commute with **all** other square matrices.

Problem 9 Friedberg, Insel and Spence 5th edition, §1.2#19, page 16.
(check with my book to make sure you're doing the right problem)

Problem 10 Let $W = \{(x, y) \mid x + y \geq 0\}$. Prove or disprove that W is a subspace of $\mathbb{R}^2$.

Problem 11 Let $W = \{(x_2 - x_4, x_2, x_4, x_4) \mid x_2, x_4 \in \mathbb{Q}\}$. Prove or disprove that W is a subspace of $\mathbb{Q}^4$.

Problem 12 A square matrix is said to be **antisymmetric** if $A^T = -A$. Let W be the set of all antisymetric $n \times n$ matrices over a field $\mathbb{F}$. Show $W \leq \mathbb{F}^{n \times n}$.

Problem 13 Let $W = \{f(t) \in \mathbb{R}[t] \mid f''(3) = 1\}$. Prove or disprove that W is a subspace of real polynomials in t.

Problem 14 Friedberg, Insel and Spence 5th edition, §1.3#12, page 21.
(check with my book to make sure you're doing the right problem)

Problem 15 Friedberg, Insel and Spence 5th edition, §1.3#13, page 21.
(check with my book to make sure you're doing the right problem)

Problem 16 Friedberg, Insel and Spence 5th edition, §1.3#15, page 22.
(check with my book to make sure you're doing the right problem)