Please follow the format which was announced in Blackboard. Thanks!
Your PRINTED NAME indicates you have read through Chapter 3 of the notes: ____________________.

Problem 19 Let $M = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 3 & 3 & 5 & 2 \end{bmatrix}$.

(a.) Calculate $\text{rref}(M)$ via Gaussian elimination over $\mathbb{R}$,

(b.) Find the solution set of $x + y + 3z = 1$ and $3x + 3y + 5z = 2$,

(c.) Find the solution set of $x_1 + x_2 + 3x_3 + x_4 = 0$ and $3x_1 + 3x_2 + 5x_3 + 2x_4 = 0$.

Remark: the problem above intends to get you to think about how a single rref matrix may have more than one use. There are many different angles we can look at a given row-reduction.

Problem 20 Suppose $[A|b] = \left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 1 & 2 & 0 & 2 \\ 1 & 3 & 1 & 0 \end{array}\right]$ is the augmented coefficient matrix of a system

$$2x + y = 1, \quad x + 2y = 2, \quad x + 3y + z = 0$$

over a field $\mathbb{F}$. Find the solution set via Gaussian elimination for:

(a.) $\mathbb{F} = \mathbb{Z}_3 = \{0, 1, 2\}$ where $2^{-1} = 2$ since $2(2) = 4 = 1$ etc.
(b.) $\mathbb{F} = \mathbb{Q}$
(c.) $\mathbb{F} = \mathbb{R}$

Problem 21 Given 2 equations in 4 unknowns over $\mathbb{Z}_5$ what are the possible sizes of the solution set?

Problem 22 Let k be a real parameter. Consider the system:

$$\begin{aligned} x + ky &= 2 \\ kx + 4y &= 1 \end{aligned}$$

Find the solution set for all three cases.

Problem 23 Let $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ over $\mathbb{R}$. Find elementary matrices $E_1, E_2, \ldots, E_k$ for which $E_k \cdots E_2 E_1 A = I$. Also, express A as a product of elementary matrices. Is A invertible?

Problem 24 Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 3 & 4 \end{bmatrix}$ over $\mathbb{R}$. Find elementary matrices $E_1, E_2, \ldots, E_k$ for which $E_k \cdots E_2 E_1 A = \text{rref}(A)$. If possible, write A as a product of elementary matrices. If not, explain why it is not possible.

Problem 25 Suppose $A \in \mathbb{R}^{4\times 4}$ such that vectors v_1, v_2, v_3, v_4 give

$$Av_1 = e_1 + e_2 + e_3 + e_4, \qquad Av_2 = e_2 + e_3 + e_4, \qquad Av_3 = e_3 + e_4, \qquad Av_4 = e_4.$$

Is A invertible? If so, find a formula for A^{-1} based on the given vectors v_1, v_2, v_3, v_4.

Problem 26 Let $A = \begin{bmatrix} 2 & 2 & 3 \\ 4 & 5 & 7 \\ 0 & 1 & 2 \end{bmatrix}$. Calculate A^{-1}.

Problem 27 For which value of k is $\{(1,2,3),(1,0,1),(2,k,8)\}$ a linearly dependent set?

Problem 28 Either prove or disprove that $S = \{(1,2,3,4),(0,1,1,0),(0,0,1,1)\}$ is a LI set in $\mathbb{R}^4$

Problem 29 Let $v = (1,1,1,2)$ and $w = (2,2,5,-1)$.

(a.) is $v \in \text{span}\{(1,1,4,-3),(2,2,8,-6)\}$?

(b.) is $w \in \text{span}\{(1,1,4,-3),(2,2,8,-6),v\}$?

Problem 30 (feel free to use technology for the row reduction here) Let

$$S = \{(1,1,1,2,2,2),(4,0,0,0,0,4)\}$$

and

$$T = \{(1,1,0,0,6,7),(0,0,1,2,-4,-5),(3,3,3,3,3,3)\}$$

(a.) Is $S \subseteq \text{span}(T)$? If not, which vector(s) in S are **not** in span(T) ?

(b.) Is $T \subseteq \text{span}(S)$? If not, which vector(s) in T are **not** in span(S) ?

Problem 31 Consider $x + 2y + 3z = 0$ and $x - y + z = 0$. Find a linearly independent set S such that the solution set of the pair of given equations is span(S).

Problem 32 Consider $S = \{(1,2,3,4),(0,1,1,7)\}$. Find a system of equations in x_1, x_2, x_3, x_4 for which span(S) is the solution set.

Problem 33 Let $V = \text{span}\{(1,1,1),(2,4,2)\}$ and $W = \text{span}\{(3,2,2),(1,0,1)\}$. Find a LI set S such that span$(S) = V \cap W$. *Hint: it's easier to combine equations than spans.*

Problem 34 Suppose A, B are invertible matrices of the same size. Show $M = \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$ is invertible.

Problem 35 We define $I \in R^{n\times n}$ by $I_{ij} = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$. If $x \in R^{n\times 1}$ then show $Ix = x$. Then, use transposition to derive $vI = v$ for all $v \in R^{1\times n}$.

Problem 36 Let $f(x) = Ax^3 + Bx^2 + Cx + D$ be a cubic polynomial for which $f(-1) = 0$ and $f'(-1) = 3$ and $f''(-1) = 8$ and $f'''(-1) = 6$. Find the values of A, B, C, D.