MATH 321: LI, SPANNING, COORDINATES, LINEAR TRANSFORMATIONS MISSION 2 [20PTS]

Your PRINTED NAME indicates you read Chapter 2 and §3.1 of the notes: _____.

We assume $\mathbb{F}$ is a field and V, W are vector spaces over $\mathbb{F}$.

Problem 17 Let $W = \{(x+y, y+z, 2x-w, x+y+z+w) \mid x, y, z, w \in \mathbb{F}\}$. Is W a subspace of $\mathbb{F}^4$?

Problem 18 Friedberg, Insel and Spence 5th edition, §1.2#21, page 16.

Problem 19 Let V_1, V_2 be vector spaces over $\mathbb{F}$. Suppose $W_1 \leq V_1$ and $W_2 \leq V_2$.
Prove $W_1 \times W_2 \leq V_1 \times V_2$.

Problem 20 Consider $S = \{t^2+1, t^2-1, t+1, t-1\} \subseteq P_4(\mathbb{R})$.

(a.) Show S is **not** linearly independent

(b.) Find a basis for $W = span\{S\}$

Problem 21 Let $W = \{A \in \mathbb{R}^{3\times 3} \mid tr(A) = 0 \ \ \& \ \ A^T = A\}$. Show $W \leq \mathbb{R}^{3\times 3}$ and find a basis for W. What is the dimension of W ?

Problem 22 Let $W = \{(x, y, z) \in \mathbb{Z}_5^3 \mid x - y - 2z = 0\}$.

(a.) Find a basis for W.

(b.) Calculate $dim(W)$.

(c.) Calculate the cardinality of W, we denote this $|W|$.

Problem 23 Consider $V = \mathbb{C}^{2\times 2}$ as a vector space over $\mathbb{R}$. Let $W = \{A \in V \mid A^T = -iA\}$.
Find a basis for W.

Problem 24 Let $\beta = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$. Find the coordinates of $v = (a, b, c, d)$ with respect to β.

Problem 25 Let $\beta = \{1, (t-1), (t-1)^2, (t-1)^3\}$. Let $f(t) = a + bt + ct^2 + dt^3$ where $a, b, c, d \in \mathbb{R}$. Calculate $[f(t)]_\beta$. *Hint: use calculus.*

Problem 26 Consider $W = \{f(t) \in P_2(\mathbb{R}) \mid \int_0^1 f(t)dt = 0\}$. Show $W \leq P_2(\mathbb{R})$ and find a basis for W.

Problem 27 Suppose $W_1 = \{f(x) \in P_4(\mathbb{R}) \mid f(1) = 0\}$ and $W_2 = \{f(x) \in P_4(\mathbb{R}) \mid f(2) = 0\}$ and $W_3 = span\{x+1, x^4-1\}$

(a.) Find a basis for $W_1 \cap W_2$

(b.) Find a basis for $W_1 \cap W_3$.

Problem 28 Friedberg, Insel and Spence 5th edition, §1.5#13, page 42.

Problem 29 Friedberg, Insel and Spence 5th edition, §1.5#20, page 43.

Problem 30 Friedberg, Insel and Spence 5th edition, §1.6#4, page 55.

Problem 31 Let $S, T \in \mathcal{L}(V)$. Prove $S \circ T \in \mathcal{L}(V)$.

Problem 32 Friedberg, Insel and Spence 5th edition, §2.1#21, page 76.