Same rules as Homework 1.

Problem 47 Your signature below indicates you have:

(a.) I read what Cook has posted of Chapter 3 of the Lecture Notes: ____________________.

(b.) I read most of Sections 5-10 of Curtis: ____________________.

Problem 48 Prove (4.) of Theorem 3.1.13 in my Lecture notes. You are free to assume that (1.), (2.) , (3.) and the Law of Cancellation are already established facts.

Problem 49 Let $W = \{A \in \mathbb{F}^{n\times n} \mid A^T = -A\}$. Show that $W \leq \mathbb{F}^{n\times n}$. (aww man, another subspace test problem)

Problem 50 Suppose $\beta = \{v_1, \dots, v_n\}$ forms a basis for the vector space V over the field $\mathbb{F}$. Show that $[cx + y]_\beta = c[x]_\beta + [y]_\beta$ for all $x, y \in V$ and $c \in \mathbb{F}$.

Problem 51 Find a basis β for $W = \{A \in \mathbb{R}^{3\times 3} \mid A^T = -A\}$.

Also, calculate $[A]_\beta$ given $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 2 \end{bmatrix}$.

There are infinitely many correct answers here, sorry Daniel.

Problem 52 Find a basis for β for $W = \{A \in \mathbb{F}^{n\times n} \mid A^T = A\}$ as a vector space over $\mathbb{F}$. Calculate $\dim_{\mathbb{F}}(W)$. (you can set $\mathbb{F} = \mathbb{R}$ if you wish to keep it real)

Problem 53 Let $W = \{p(t) \in \mathbb{R}[t] \mid p^{(n)}(1) = 0 \text{ for } n = 3, 4, 5, \dots\}$. Prove W is a subspace of $\mathbb{R}[t]$ by explicitly finding S for which $W = \text{span}(S)$.

Problem 54 Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$. Find bases for the column, row and null space of A. For the column and row space use actual columns and rows of A to form the bases.

Problem 55 Let $A = \begin{bmatrix} \overline{1} & \overline{0} \\ \overline{2} & \overline{1} \\ \overline{9} & \overline{1} \end{bmatrix} \in (\mathbb{Z}/11\mathbb{Z}.)^{3\times 2}$. Find bases for the column, row and null space of A.

Problem 56 Consider $W = \text{span}_{\mathbb{C}}\{1, x, x^2\}$ as a vector space over $\mathbb{R}$. Find a basis for W.

Problem 57 Let $\beta = \{x^2, (1 + x)^2\}$ and define $W = \text{span}(\beta)$ as a subspace of $P_2(\mathbb{R})$. If $f(x) = ax^2 + bx + c \in W$ then find $[f(x)]_\beta$.

Problem 58 Curtis §5 #3 on page 37. Your solution should focus on the LI of the proposed basis. Prove $\{1, x, x^2, \dots, x^n\}$ is a LI set.

Problem 59 Curtis §6 #5 on page 48. (linear dependence in polynomials)

Problem 60 Curtis §7 #4 on page 52. (subspace intersection calculation)

Problem 61 Curtis §7 #5 on page 52. (subspace intersection proof question)

Problem 62 Curtis §8 #2 on page 61. (solvability of system)

Problem 63 Curtis §8 #4 on page 62 (I'd like you to use my Proposition 3.8.4 or something similar)

Problem 64 Curtis §9 #6 on page 69 (a result in the algebraic geometry of lines)

Problem 65 Curtis §10 #2 on page 73 (interesting reverse of our usual calculation)

Problem 66 Curtis §10 #7 on page 74 (hyperplanes as solution sets)

Problem 67 Curtis §10 #8 on page 74 (basis for intersection of subspace)

Problem 68 Prove Theorem 3.7.4 in my notes. In particular, show the following is true:
If V is a vector space over a field $\mathbb{F}$ and $U \leq V$ and $W \leq V$ then $U \cap W \leq V$

Problem 69 Prove Theorem 3.7.6 in my notes. In particular, show the following is true:
If V is a vector space over a field $\mathbb{F}$ and $U \leq V$ and $W \leq V$ then $U + W \leq V$.
Furthermore, $U + W$ is the smallest subspace which contains $U \cup W$.

Problem 70 Prove Proposition 3.4.4 of the Lecture Notes: that is, prove the following:
Let V be a vector space over a field $\mathbb{F}$ and $S \subseteq V$. S is a linearly independent set of vectors iff if there exist $a_i, b_i \in \mathbb{F}$ for $i \in \mathbb{N}_k$ for which

$$a_1 v_1 + a_2 v_2 + \cdots + a_k v_k = b_1 v_1 + b_2 v_2 + \cdots + b_k v_k$$

then $a_i = b_i$ for each $i = 1, 2, \ldots, k$. In other words, we can equate coefficients of a set of vectors iff the set of vectors is a LI set.