Same rules as Homework 1.

Problem 91 Your signature below indicates you have:

(a.) I read Chapter 5 of Cook's lecture notes: ____________________.

Problem 92 Remember, we have many properties to use in addition to the cofactor formulae,

(a.) Calculate $\det(A)$ where $A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 5 & 3 & 1 \end{bmatrix}$

(b.) Calculate $\det(B)$ where $B = \begin{bmatrix} 2 & 2 & 0 & 2 & 2 \\ 0 & 2 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 & 3 \\ 7 & 7 & 2 & 7 & 7 \\ 5 & 3 & 0 & 0 & 0 \end{bmatrix}$

(c.) Let A, B be as given in the previous problems. If $M = \left[\begin{array}{c|c} 2A & 0 \\ \hline 0 & 3B \end{array}\right]$ then calculate $\det(M)$ via application of properties of determinants given in the lecture notes and the results of the previous pair of problems.

Problem 93 For which values of x is the matrix $M = \begin{bmatrix} x & 2 & 2 \\ 1 & 1 & 1 \\ 7 & 5 & 3 \end{bmatrix}$ invertible?

Problem 94 **Square root of a matrix?**

(a.) Consider the problem $A^2 = X$ if there is a matrix A for which this matrix equation has a solution then it would be reasonable to think of $A = \sqrt{X}$. Consider, if $\det(X) = -1$ is it possible to find a real matrix A for which $A^2 = X$?

(b.) Consider $X = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$. Find a real matrix A for which $A^2 = X$.

Problem 95 Solve $Ax + 3y = 7$ and $5x - By = 6$ by Cramer's rule. Comment on needed conditions on A, B for the solution to exist.

Problem 96 Let A be a matrix which is similar to $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. In other words, suppose there exists an invertible matrix P for which $B = P^{-1}AP$. Calculate $\det(A)$ and $\text{trace}(A)$.

Problem 97 Calculate $\det \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{bmatrix}$ by row reduction paired with the identities derived in the notes for elementary row operations.

Problem 98 Determinant calculates volume

(a.) Find the volume of the paralell piped with edges $(1, 2, 3), (2, 3, 3), (-1, -2, 0)$.

(b.) Let P be a paralell piped with edges $u, v, w \in \mathbb{R}^3$ and let $T : \mathbb{R}^3 \to \mathbb{R}^3$. Show that $\text{Vol}(T(P)) = \det([T])\text{Vol}(P)$. Use the concatenation proposition! This problem need not be difficult.

Problem 99 The cross product: For all $a, b, c \in \mathbb{R}^3$ we define

$$T(a, b) = \sum_{j=1}^{3} \big(\det[a|b|e_j]\big) e_j.$$

Show $T(a, b) \bullet c = \det[a|b|c]$ and thus $T(a, b) = -T(b, a)$ and $a \bullet T(a, b) = 0$.

Problem 100 A natural candidate for the cross product in $\mathbb{R}^4$ is given by extending the formula in the previous problem: for all $a, b, c \in \mathbb{R}^4$ we define

$$T(a, b, c) = \sum_{j=1}^{4} \big(\det[a|b|c|e_j]\big) e_j$$

Show: Show $T(a, b, c) \bullet d = \det[a|b|c|d]$. Use this identity to help prove the **claim**:
Claim: if $\{a, b, c\}$ is LI then $\{a, b, c, T(a, b, c)\}$ is LI.

Note: I think Problem 100 is easily as hard as the rest of these problems combined.