Same rules as Homework 1. However, do keep in mind you are free to use technology to find e-vectors for 3×3 examples. Also, it would be wise to work out at least one of them by hand to increase your skill-level. Usually, students need some practice with factoring characteristic equations (actually, the most critical difficulty here is not finding $\text{char}(x) = \det(x - T)$, but, in the polynomial factoring which follows...)

Problem 101 Your signature below indicates you have:

(a.) I read Chapter 7 of Curtis: ____________________.

(b.) I read Chapter 7 supplemental by Cook: ____________________.

(c.) I watched the extra magic hour Lectures: ____________________.

Problem 102 Curtis §22 exercise #11 on page 193.

Problem 103 Curtis §23 exercise #3 on page 201.

Problem 104 Curtis §23 exercise #6 on page 201.

Problem 105 Curtis §23 exercise #7 on page 201.

Problem 106 Curtis §24 exercise #5 on page 215.

Problem 107 Curtis §24 exercise #8 on page 216.

Problem 108 Curtis §25 exercise #6 on page 226.

Problem 109 Let $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$. I'll be nice and tell you the eigenvalues are $\alpha = 1, -2$ and 3. Find an eigenbasis with respect to A. Diagonalize A and calculate A^n explicitly.

Problem 110 Continuing the previous problem, let $T = L_A$ and find the standard matrices of transformations E_1, E_2, E_3 for which $E_j\mathbb{R}^3 = \text{Ker}(T - \alpha_j)$ and $\mathbb{R}^3 = E_1\mathbb{R}^3 \oplus E_2\mathbb{R}^3 \oplus E_3\mathbb{R}^3$ where E_1, E_2, E_3 are idempotent and pairwise commuting with $E_1E_2 = E_1E_3 = E_2E_3 = 0$.

Problem 111 Consider $V = \text{span}_{\mathbb{R}}\{\cosh(x), \sinh(x), \cos(x), \sin(x)\}$ and let $T = D^2 + 1$ where $D = d/dx$. Find the eigenvalues of T and decompose V as the direct sum of eigenspaces.

Problem 112 Consider $V = \text{span}_{\mathbb{R}}\{\cosh(x), \sinh(x), \cos(x), \sin(x)\}$ and let $T = D$ where $D = d/dx$. Find the characteristic and minimal polynomials of T. Also, find the rational cannonical form of T.

Problem 113 Consider T of the previous problem once more. Complexify T and find a complex eigenbasis for the complexification of T. Also, determine the real Jordan form of T.

Problem 114 Find the companion matrix of $p(x) = x^5 + 3x^4 + 2x^2 - 3x - 9$ and $p(x)^2$ using the set-up as in Curtis' Chapter 7 (while prime polynomials are of primary interest to the classification results of Chapter 7, we can just as well calculate the companion matrix for any polynomial).

Problem 115 Suppose you are given a 5×5 matrix C with eigenvalues $\lambda = 2$ repeated five times. List the possible Jordan forms which are similar to the given C; that is $A = P^{-1}CP$ with $P = [\beta]$ where β is a Jordan basis for C. For each possibility, also find the minimal polynomial of C. To keep your list shorter, we'll consider two matrices with the same set of diagonal blocks the same Jordan form.

Problem 116 Suppose you are given a real 4×4 matrix C with complex eigenvalues $\lambda = 2 + 3i$ repeated twice. List the possible real Jordan forms which are similar to the given C. For each possibility, also find the minimal polynomial C. To keep your list shorter, we'll consider two matrices with the same set of diagonal blocks the same real Jordan form.

Problem 117 Suppose that If $A = \mathrm{diag}\left(\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \right)$ where this notation indicates that A is block-diagonal with the diagonal blocks as given. Find the eigenvalues of A and state the algebraic and geometric multiplicity of each eigenvalue. Recall we use the notation λ_j has algebraic multiplicity a_j and geometric multiplicity g_j.

Problem 118 Let V be a real 4 dimensional vector space. Suppose $T : V \to V$ is a linear transformation such that:

$$T(v_1) = v_1, \quad T(v_2) = 3v_2, \quad T(v_3) = 6v_3 - 7v_4, \quad T(v_4) = 6v_4 + 7v_3$$

Find the eigenvalues and complex eigenvalues of T. (technically, complex eigenvalues are the eigenvalues of the complexification of T)

Problem 119 Find the general solution of $\frac{d\vec{x}}{dt} = A\vec{x}$ where A is as was given in Problem 102. Also, explicitly find the matrix exponential e^{tA}.

Problem 120 Suppose T is a real linear transformation on a vector space with basis $\vec{a}, \vec{b}$. Also, suppose the complexification of T has $T(\vec{u}) = (3 + 2i)\vec{u}$ where $\vec{u} = \vec{a} + i\vec{b}$ for real vectors $\vec{a}, \vec{b}$. Find the general real solution of $\frac{d\vec{v}}{dt} = T(\vec{v})$.