We assume $\mathbb{F}$ is a field and V, W are vector spaces over $\mathbb{F}$.

Problem 1 Let A, B, N, S be invertible square matrices. Solve the following equation for X:

$$(BA)^{-1}XS^{-1} = (NA)^2$$

Problem 2 Let V_1, V_2 be subspaces of V over $\mathbb{F}$. Prove $V_1 \cap V_2$ is a subspace of V.

Problem 3 You are given $T : \mathbb{R}^3 \to \mathbb{R}^3$ has $T(e_1) = (0, 1, 1)$ and $T(e_2) = (1, 2, 3)$ and $T(e_3) = (1, 1, 2)$. Find the bases for the kernel and image of T. Is T an surjection ? Is T an injection ? Is T an isomorphism ?

Problem 4 Friedberg, Insel and Spence 5th edition, §1.6#17, page 56.

Problem 5 Friedberg, Insel and Spence 5th edition, §2.1#5, page 74.

Problem 6 Friedberg, Insel and Spence 5th edition, §2.1#15, page 75.

Problem 7 Friedberg, Insel and Spence 5th edition, §2.1#17, page 76.

Problem 8 Let $T(f(x)) = \begin{bmatrix} f(0) & if'(0) \\ if(1) & f'(1) \end{bmatrix}$ define a linear transformation from $V = P_2(\mathbb{R})$ to $W = \mathbb{C}^{2\times 2}$. If we use basis $\beta = \{1, x, x^2\}$ for $V(\mathbb{R})$ and $\gamma = \{E_{11}, iE_{11}, E_{12}, iE_{12}, E_{21}, iE_{21}, E_{22}, iE_{22}\}$ for $W(\mathbb{R})$ then find $[T]_{\beta,\gamma}$. Is T an injective map ?

Problem 9 Define $T : P_3(\mathbb{R}) \to \mathbb{R}^{1\times 3}$ by $T(f(x)) = [f(1), f(2), f(1) + f(2)]$. Find a basis β for $P_3(\mathbb{R})$ and γ for $\mathbb{R}^{1\times 3}$ for which $[T]_{\beta,\gamma} = \left[\begin{array}{c|c} I_p & 0 \\ \hline 0 & 0 \end{array}\right]$ where $p = rank(T)$.

Problem 10 Let $A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 2 & -1 & 3 \\ 1 & 3 & -2 & 4 \end{bmatrix}$. If $\beta = \{1, x, x^2, x^3\} \subseteq P_3(\mathbb{R})$ and $\gamma = \{E_{11}, E_{22}, E_{12} + E_{21}\} \subseteq \mathbb{R}^{2\times 2}$ and $[T]_{\beta,\gamma} = A$ then find the formula for $T(a + bx + cx^2 + dx^3)$ and find the basis for $Ker(T)$.

Problem 11 Suppose $L : P_2(\mathbb{R}) \to \mathbb{R}^{2\times 2}$ is a linear transformation for which

$$L(1) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \& \quad L(t) = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} \quad \& \quad L(t^2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

. Let $\beta = \{t^2, t, 1\}$ and $\gamma = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ serve as bases for $P_2(\mathbb{R})$ and $\mathbb{R}^{2\times 2}$ respective.

(a.) find $[T]_{\beta,\gamma}$

(b.) calculate the rank and nullity of T

(c.) find a basis for $Ker(T)$

Problem 12 Suppose $W_1 \leq W_2 \leq V$ over $\mathbb{F}$. Prove $\text{ann}(W_2) \leq \text{ann}(W_1) \leq V^*$.

Problem 13 Friedberg, Insel and Spence 5th edition, §2.6#15, page 127.

Problem 14 Let $V = P_4(\mathbb{Q}) \times \mathbb{Q}^{2\times 2}$. Find an isomorphism of V and $\mathbb{Q}^{3\times 3}$.

Problem 15 Find an isomorphism of $V = \{A \in \mathbb{R}^{3\times 3} \mid A^T = A\}$ to $\mathbb{C}^n$ for an appropriate choice of n.

Problem 16 (Bonus) Friedberg, Insel and Spence 5th edition, §1.7#3, page 62.