# Linear Algebra Test I Overview

As always your first best line of defense is to complete and understand the homework, Problem Set, and lecture examples.

**Calculational Skills:**

1. Gauss-Jordon Elimination (see definition 1.2.1. although I am flexible on how you do the middle of the method. You do not have to to the forward and backwards pass separately if you prefer mixing it up... however, we should all agree on the end result since $rref(A)$ is unique for a given matrix $A$.)
2. Be able to find a polynomial to fit a given set of points, or be able to fit a model with linear coefficients to a given set of data. We saw this in Examples 1.4.1,1.4.2 and Problems 8 and 9 in the Problem Set I.
3. Be able to solve problems similar to the traffic flow example.
4. Multiply, add and subtract matrices. Also know how to multiply a matrix by a number.
5. Be able to extract scalar equations from a matrix equation and vice-versa.
6. Know your 2x2 inverse formula so you can calculate an inverse of a 2x2 matrix without too much thinking.
7. Know the algorithm to find the inverse for a $3x3$ or larger matrix.
8. I do **not** expect you to know the definition for the adjoint or Kramer's Rule for the exam. Those are calculationally inferior methods for most numerical problems.
9. Be able to answer questions about system governed by a stochastic matrix. This would be like Example 2.10.2 but I would have nice numbers so multiplication without a calculator is reasonable.
10. Know how to extract the general solution from a given $rref(A|b)$.

**Terms you should know the meaning and be able to give examples of:**

1. unique system of linear equations
2. homogeneous system of linear equations
3. inconsistent system of linear equations
4. general solution for a system of linear equations
5. augmented coefficient matrix for a linear system of equations
6. coeffient matrix for a system of linear equations
7. basic variables
8. free variables
9. concatenation of matrices

**Definitions you should be able to state precisely:**

1. system of linear equations (Defn. 1.1.5)
2. homogeneous system of linear equations (Defn. 1.1.5)

3. solution set of a system of linear equations (Defn. 1.1.5)
4. augmented coefficient matrix for a given system of linear equations (Defn. 1.1.7)
5. the matrix form of a system of linear equations (see Prop. 2.6.1, the definition is actually part of that prop. The matrix form of the system is simply $Ax = b$ given the notation of that proposition)
6. matrix equality (Defn. 2.1.3)
7. rows and columns of a matrix (Defn. 2.1.4)
8. transpose of a matrix (Defn. 2.1.6)
9. matrix addition (Defn. 2.2.1)
10. scalar multiple of a matrix (Defn. 2.2.3)
11. the zero matrix (Defn. 2.2.6)
12. matrix multiplication (Defn. 2.3.1)
13. the identity matrix (Defn. 2.3.7)
14. elementary matrices (Defn. 2.4.1)
15. inverse matrix (Defn. 2.5.1)
16. standard basis $e_i$ (Defn. 2.8.2)
17. standard unit matrices $E_{ij}$ (Defn. 2.8.5)
18. symmetric matrices (Defn. 2.9.1)
19. antisymmetric matrices (Defn. 2.9.1)
20. integer powers of a square matrix (Defn. 2.9.8)

It is also assumed you know the terms from prerequisite courses.

**Propositions which contain properties you should use unless otherwise instructed:**

1. Prop. 1.2.9 is at times useful.
2. Theorem 1.5.1 gives us the big picture.
3. Prop. 2.3.11 the concatenation proposition, a calculationally important observation. I am fond of this one, I don't deny it.
4. Prop. 2.3.13 use these to help with matrix calculations.
5. Prop. 2.4.3 says $rref(A) = E_1 E_2 \cdots E_k A$ where $E_i$ are elementary matrices. Mostly a useful observation for proofs.
6. Prop. 2.5.5 says you can just check the left or just the right inverse condition.
7. Prop. 2.5.9 items 1 and 2.
8. Prop. 2.5.11 multiple socks-shoes identity.
9. Prop. 2.9.3 properties of transpose

10. Prop. 2.9.9 properties of matrix powers, careful not to extend this past what it actually says. For example, for matrices it is not usually true that $(A+B)^2 = A^2 + 2AB + B^2$ (this is false since $AB \neq BA$ in general)

11. Theorems 3.8.1 and 3.8.2 the big picture theorems.

12. Theorem 3.2.4, the cofactor theorem for the determinant. You should be able to expand to maximize the ease of you calculation.

13. Prop 3.5.1 properties of determinants.

14. Prop 3.5.6 says $det(AB) = det(A)det(B)$.

15. Proof of Theorem 2.5.4, see Problem 16 solution from Problem Set 1. There is an idea about constructing the inverse from elementary matrices which can be useful in certain instances where explicit calculation of $rref[A|I]$ is too cumbersome.

**Type of "Proofs" you might encounter on the test:**

1. Problems 5,6,7,15,17,18,21b,22,25,26,30,21,32,33,34 of Problem Set 1.

2. Theorem 2.5.4 ( I would allow you to assume that elementary matrices are invertible and $rref(A) = E_1E_2 \cdots E_kA$ for some list of elmentary matrices and also the little Theorem from Chapter 1 which says that a unique solution for $Ax = b$ implies $rref[A|b] = [I|x]$.

3. Theorem 2.2.8.

I will not ask the harder theorems from Chapter 3 and generally speaking I will tend towards proofs which require a minimum of background theorems. If Theorems/Propositions are needed then I will probably supply them in the statement of the problem. For example, if I was to ask for a proof of Theorem 2.5.4 I would remind you the facts that went into the proof.

**Tentative breakdown of the 1500pts on the exam: (not necessarily in this order)**

1. maybe 5 definitions of my choosing (100pts).

2. solve some system by row operations and multiplication by inverse (same problem two ways) (100pts)

3. find the inverse of a 3x3 or 4x4 matrix (150pts)

4. an application problem (150pts)

5. Given $rref(A|b)$ for some system $Ax = b$ find the general solution, might need to write answer in "parametric form" in terms of "free variables" (100pts)

6. Matrix calculations of an explicit or conceptual nature (200pts)

7. a determinant question (100pts)

8. conceptual question(s), perhaps true/false give counter example. Also would be designed to see if you absorbed the ideas of the main theorems in the first three chapters, particularly Theorem 3.8.1 and 3.8.2 (200pts)

9. "proofs" (400pts)

Tuesday 7-8pm in Science Hall 105 (the Physics Lab) I will host a "review". This means I will answer questions about this review. Of course you can also ask questions in office hours Monday from 5-6:20pm.