Math 321 Test 1

Please show your work and use words to explain your steps where appropriate.

Notational Conventions for Test: V and W denote vector spaces over a field  $\mathbb{F}$ . Also, L(V,W) is the set of linear transformations from V to W.  $\mathbb{R}[x]$  is the set of real polynomials.  $P_n(\mathbb{F}) = span_{\mathbb{F}}\{1, x, x^2, \dots, x^n\}$ . Also,  $S \leq V$  means S is a subspace of V. LI means Linearly Independent

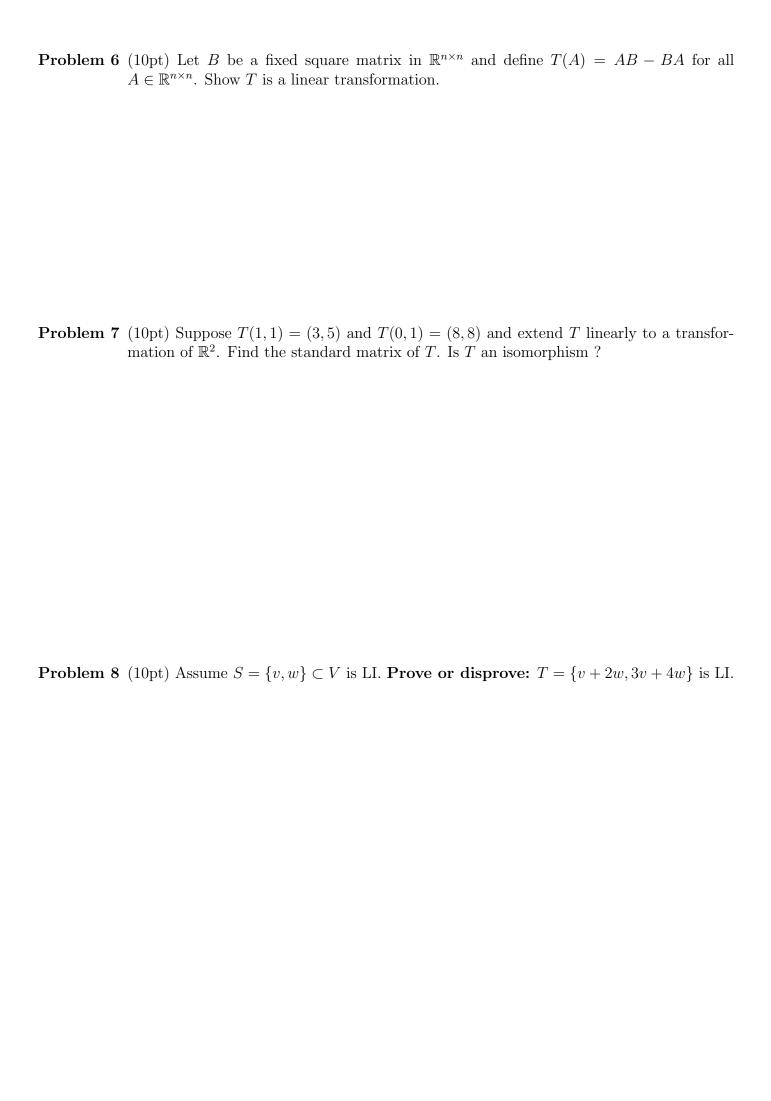
**Problem 1** (5pt) Define what is meant by the statement:  $\beta$  is a basis for V (assume  $V \neq 0$ )

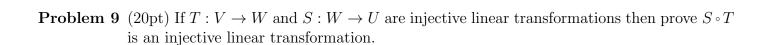
**Problem 2** (10pt)  $P_2(\mathbb{R})$  has basis  $\beta = \{(x+1)^2, x+1, 1\}$ . If  $f(x) = 8x^2 + 6x + 21$  then find  $[f(x)]_{\beta}$ .

**Problem 3** (5pt) Is  $\mathbb{Q}$  a subspace of  $V = \mathbb{R}$  where V is a real vector space. ?

**Problem 4** (10pt) Let  $W = \{ax^2 + bx^2 + ax + b \mid a, b \in \mathbb{C}\}$ . Prove  $W \leq \mathbb{C}[x]$ .

**Problem 5** (10pt) Let  $T: V \to W$  be a linear transformation. Prove  $Ker(T) \leq V$ .





**Problem 10** (5pt) A linear manifold has the same dimension as its directing space;  $\mathcal{M} = p + \mathcal{S}$  then  $\dim \mathcal{M} = \dim \mathcal{S}$ . If  $\mathcal{S} = \operatorname{span}\{v_1, v_2, v_3, v_4\}$  where  $\mathcal{S} \leq V$  and  $\dim(V) = 3$  then what are the possible dimensions of  $\mathcal{M}$ ?

**Problem 11** (15pt) Let  $T(f(x)) = \begin{bmatrix} f(0) & f'(0) \\ f'(0) & 0 \end{bmatrix}$  for each  $f(x) \in P_2(\mathbb{R})$ . Note  $T : P_2(\mathbb{R}) \to \mathbb{R}^{2 \times 2}$ . Let  $P_2(\mathbb{R})$  have basis  $\beta = \{1, x, x^2\}$  and  $\mathbb{R}^{2 \times 2}$  have basis  $\gamma = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ . Find  $[T]_{\beta\gamma}$ 



