MATH 331 QUIZ 2

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps.

Problem 1 Let $a, b > 0$ and calculate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$

Problem 2 Calculate $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x dx}{(x+\alpha)^2+\beta^2}$

Problem 3 Calculate $\displaystyle\int_{0}^{\infty} \frac{\ln(x)\,dx}{1+x^2}$.

Problem 4 Let $n \in \mathbb{N}$. Show that if $P(z) = (z-z_1)^{m_1}(z-z_2)^{m_2}\cdots(z-z_n)^{m_n}$ then

$$\frac{P'(z)}{P(z)} = \frac{m_1}{z-z_1} + \frac{m_2}{z-z_2} + \cdots + \frac{m_n}{z-z_n}.$$

Problem 5 Suppose $P(z)$ is a polynomial with no zeros along the simple closed contour Γ (this means $\Gamma = \partial D$ where D is a simply connected subset of $\mathbb{C}$) then show that the number of zeros inside Γ, counting multiplicities, is given by:

$$\frac{1}{2\pi i}\int_{\Gamma} \frac{P'(z)}{P(z)} dz.$$

Problem 6 Determine the number of zeros of $p(z) = z^9 - 2z^6 + z^2 - 8z - 2$ inside the unit-circle $|z| = 1$.

Problem 7 Saff and Snider §6.2#8 (integral of trigonmetric function)

Problem 8 Saff and Snider §6.3#2 (infinite contour integral)

Problem 9 Saff and Snider §6.4#9 (Jordan Lemma type infinite contour integral)

Problem 10 Saff and Snider §6.5#5 (indented contour integral)

Problem 11 Saff and Snider §6.6#1 (integral involving branch cut)

Problem 12 Saff and Snider §6.6#4 (integral involving branch cut)

Problem 13 Saff and Snider §6.7#1 (meromorphic function trivia)