Same instruction as Mission 1. Enjoy !

Problem 31 Suppose f is holomorphic on a domain D and $f(D) \subseteq i\mathbb{R}$. Prove f is constant.
Hint: see Theorem 5.3.9 which asserts a similar result in the context $f(D) \subseteq \mathbb{R}$

Problem 32 Let f be complex differentiable at $g(z_o)$ and let g be complex differentiable at z_o. Prove the chain rule of complex calculus; that is, show $(f \circ g)'(z_o) = f'(g(z_o))g'(z_o)$.

Problem 33 See Definitions 2.3.4 and 2.3.5 for the precise meaning of $\mathbb{C}^\alpha$ and $Log_\alpha(z)$. Show that $Log_\alpha \in \mathcal{O}(\mathbb{C}^\alpha)$ with $\frac{d}{dz}Log_\alpha(z) = \frac{1}{z}$ for each $z \in \mathbb{C}^\alpha$. Explain why Log_α is not continuous on $\mathbb{C}^\times = \mathbb{C} - \{0\}$. Why is Log_α not holomorphic on the punctured plane ? Discuss.

Problem 34 Let $f(z) = \frac{Log(z+5)}{z^2+3z+2}$. Show $f(z)$ is holomorphic everywhere except the points $-1, -2$ and the ray $(-\infty, -5] = \{x + iy \mid x \leq -5, y = 0\}$.

Problem 35 Saff and Snider §2.5#3. (calculation of harmonic conjugates)

Problem 36 Saff and Snider §3.1#13. (partial fractions)

Problem 37 Saff and Snider §3.1#17. (log derivative identity)

Problem 38 Saff and Snider §3.2#14. (periodicity of exponential, tangent and hyperbolic functions)

Problem 39 Saff and Snider §3.3#11. (find a branch for a log of a quadratic)

Problem 40 Saff and Snider §3.4#4. (solve Laplace's equation on strip)

Problem 41 Saff and Snider §3.5#6. (identities for principal branch of power function)

Problem 42 Saff and Snider §3.5#8. (solve sine equation)

Problem 43 Saff and Snider §3.5#14. (derivative of inverse hyperbolic sinh)

Problem 44 Let $u(z) = (x^2 - y^2)e^x \cos y - 2xye^x \sin y$ for each $z = x + iy \in \mathbb{C}$. A wandering mathematician crosses your path and tells you to consider the square and exponential. Find the harmonic conjugate of u for which $u + iv$ is holomorphic on $\mathbb{C}$.

Problem 45 Saff and Snider §7.1#2. (Laplace equation transfer via gradient analysis)