Math 331 Mission 5

Same instruction as Mission 1. Enjoy!

- **Problem 61** Saff and Snider §4.3#1 (integration of many interesting functions along funny contour)
- **Problem 62** Saff and Snider §4.4#10 (integration of nice functions on a circle)
- **Problem 63** Saff and Snider §4.4#17 (partial fractions integration)
- **Problem 64** Saff and Snider §4.5#3 (integration based on Cauchy's ∫-formula)
- **Problem 65** Saff and Snider §4.5#16 (setting up single-valued branch of log(f(z)))
- **Problem 66** Saff and Snider §5.2#3 (using a power series to form new series)
- **Problem 67** Saff and Snider §5.2#8 (proof via Taylor series argument)
- **Problem 68** Saff and Snider §5.2#11 (first few non-trivial terms via multiplication)
- **Problem 69** Saff and Snider §5.3#1 (convergence on boundary, nice examples)
- **Problem 70** Saff and Snider §5.3#3 (ratio test for finding ROC for power series)
- **Problem 71** Saff and Snider §5.3#14 (definition of sine via differential equation)
- **Problem 72** Saff and Snider §5.4#12 (Riemann zeta function analytic for Re(z) > 1)
- **Problem 73** Find the power series expansion of f(z) = Log(z) about $z_o = i 2$ and show the R.O.C. is $R = \sqrt{5}$. We know Log(z) is discontinuous at z = -2, is the power series centered at z_o discontinuous at z = -2? Discuss the relation of the power series centered at z_o with the given function.
- **Problem 74** Let $\alpha \in \mathbb{C}$ and let $f(z) = (1+z)^{\alpha}$ denote the principle branch of the α -power function. Derive the Taylor series for f(z) centered at $z_o = 0$ and calculate the radius of convergence for your series.
- **Problem 75** Show that if $f(z) = \frac{A_k}{z^k} + \frac{A_{k-1}}{z^{k-1}} + \dots + \frac{A_1}{z} + g(z)$ where g(z) is holomorphic on $|z| \le 1$ then $\oint_{|z|=1} f(z)dz = 2\pi i A_1$.

 you feel something inside you start to stir. An instinct begins to arise...