Same instructions as Tour 1. Thanks!

Problem 49 Consider $F : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $F(r,\theta,\phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$. View r,θ,ϕ as Cartesian coordinates on the domain of F and view x,y,z as coordinates for the codomain of F. In particular, this indicates $F^1 = x$ and $F^2 = y$ and $F^3 = z$. Find $F^*(\beta)$ where $\beta = \frac{1}{x^2+y^2}[-ydx + xdy]$.

Problem 50 (Lee p.389 Problem 11) Let ω below defined for $(x,y,z) \neq 0$,

$$\omega = \frac{x\,dy\wedge dz + y\,dz\wedge dx + z\,dx\wedge dy}{(x^2+y^2+z^2)^{3/2}}$$

Calculate $d\omega$. Is ω closed? Is ω exact? Express ω in spherical coordinates (pull-back dx, dy, dz etc. to $d\rho, d\phi, d\theta$ etc.)

Problem 51 Renteln Exercises 3.32 and 3.33 page 97. (pull-backs)

Problem 52 Renteln Exercise 3.35 page 98. (relation of push-forward to Jacobian matrix)

Problem 53 Let U be open subset of $\mathbb{R}^n$. Suppose ω_1 is a closed differential form on $U \subseteq \mathbb{R}^n$ and suppose ω_2 is an exact differential form on $U \subseteq \mathbb{R}^n$. Show $\omega_1 \wedge \omega_2$ is exact.

Problem 54 Problem from Lecture to be added here.

Problem 55 Problem from Lecture to be added here.

Problem 56 Problem from Lecture to be added here.

Problem 57 Problem from Lecture to be added here.

Problem 58 Problem from Lecture to be added here.

Problem 59 Problem from Lecture to be added here.

Problem 60 Problem from Lecture to be added here.

Problem 61 Renteln Exercise 6.3 page 166. (verify Stokes' theorem)

Problem 62 Renteln Exercise 6.4 page 166. (integral of exact or closed)

Problem 63 Renteln Exercise 6.5 page 166. (connection between d^2 and ∂^2)

Problem 64 Exercise 6.8 and 6.9 on page 171-172 of Renteln (a natural homomorphism powered by IBP on forms)