I will select some subset of these problems to collect. The more you work, the more you know.

**PP 1** (Separation of Variables) Solve the differential equations below. If possible, find the explicit solution, otherwise find an implicit general solution.

**(a.)** $\dfrac{dy}{dx} = (x+1)^2$

**(b.)** $dx - x^2 dy = 0$

**(c.)** $e^x \dfrac{dy}{dx} = 2x$

**(d.)** $\dfrac{dy}{dt} + 2xt = 0$

**(e.)** $\dfrac{dy}{dx} = \dfrac{y+1}{x}$

**(f.)** $\dfrac{dx}{dy} = \dfrac{1+2y^2}{y \sin x}$

**(g.)** $x^2 y^2 dy = (y+1)dx$

**(h.)** $\dfrac{dy}{dx} = \left(\dfrac{2y+3}{4x+5}\right)^2$

**(i.)** $\sec x dy = x \cot y dx$

**(j.)** $x\sqrt{1-z^2}dx = dz$

**(k.)** $\sec y \dfrac{dy}{dx} + \sin(x-y) = \sin(x+y)$

**(l.)** $\dfrac{dy}{dx} = \dfrac{xy+3x-y-3}{xy-2x+4y-8}$

**(m.)** $(e^x + e^{-x})\dfrac{dw}{dx} = w^2$

**(n.)** $(x+\sqrt{x})\dfrac{dy}{dx} = y + \sqrt{y}$.

**PP 2** (Initial Value Problems) Use separation of variables to solve the IVPs below:

**(a.)** $\dfrac{dy}{dt} + ty = y$ with $y(1) = 3$.

**(b.)** $\dfrac{dx}{dy} = 4(x^2+1)$ with $x(\pi/4) = 1$.

**(c.)** $\dfrac{dy}{dx} = \dfrac{y^2-1}{x^2-1}$ with $y(2) = 2$

**(d.)** $y' + 2y = 1$ with $y(0) = 5/2$.

**PP 3** (substitution of form $u = ax + by + c$) Solve the following by making an appropriate substitution and using separation of variables,

**(a.)** $\frac{dy}{dx} = (x + y + 1)^2$

**(b.)** $\frac{dy}{dx} = \tan^2(x + y)$

**(c.)** $\frac{dy}{dx} = \frac{1 - x - y}{x + y}$

**(d.)** $\frac{dy}{dx} = 1 + e^{y-x+5}$

**PP 4** (homogeneous equations, try $y = ux$ or $x = vy$ on $M(x, y)dx + N(x, y)dy = 0$ where $M$ and $N$ are homogeneous functions of same degree)

**(a.)** $(x - y)dx + xdy = 0$

**(b.)** $\frac{dy}{dx} = \frac{y - x}{y + x}$

**(c.)** $y\frac{dx}{dy} = x + 4ye^{-2x/y}$

**(d.)** $(x^2 + xy - y^2)dx + xydy = 0$

**PP 5** (exact equations) If the DEqn below is exact then solve it, otherwise explain why the given DEqn is not exact.

**(a.)** $(2x + y)dx - (x + 6y)dy = 0$

**(b.)** $(2xz^2 - 3)dx + (2zx^2 + 4)dz = 0$

**(c.)** $(\sin y - y \sin x)dx + (\cos x + x \cos y - y)dy = 0$

**(d.)** $\left(2y - \frac{1}{x} + \cos 3x\right)\frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y \sin 3x = 0$

**(e.)** $\left(1 + \ln x + \frac{y}{x}\right)dx = (1 - \ln x)dy$

**(f.)** $(\theta^3 + \beta^3)d\theta + 3\theta\beta^2 d\beta = 0$

**(g.)** $(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$

**(h.)** $(e^y + 2xy \cosh x)y' + xy^2 \sinh x + y^2 \cosh x = 0$

**(i.)** $(2y \sin x \cos x - y + 2y^2e^{xy^2})dx = (x - \sin^2 x - 4xye^{xy^2})dy$

**(j.)** $\left(\frac{1}{x} + \frac{1}{x^2} - \frac{t}{x^2 + t^2}\right)dx + \left(te^t + \frac{x}{x^2 + t^2}\right)dt = 0$

**PP 6** (exact equations with integrating factor) A general form of an integrating factor is suggested. Find the specific form $I$ which serves as an integrating factor and solve the DEqn $Mdx + Ndy = 0$ by solving the exact equation $IMdx + INdy = 0$)

**(a.)** $6xydx + (4y + 9x^2)dy = 0$ given $I = y^A$

**(b.)** $y(x + y + 1)dx + (x + 2y)dy = 0$ given $I = e^{Ax}$

**(c.)** $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$ given $I = (x + y)^A$

**(d.)** $y(4xy^5 + 3)dx - x(2xy^5 + 7)dy = 0$ given $I = x^A y^B$

**PP 7** (linear first order DEqn) Solve the linear first order ODEqn given below and state the interval on which the solution is defined. If given an initial value, then fit the given data to the explicit solution.

**(a.)** $\dfrac{dy}{dx} + y = e^{3x}$

**(b.)** $y' + 3x^2 y = x^2$

**(c.)** $\dfrac{dx}{dy} = x + y$

**(d.)** $x\dfrac{dy}{dx} + 2y = 3$

**(e.)** $xdy = x \sin x - y)dx$

**(f.)** $x\dfrac{dy}{dx} + 4y = x^3 - x$

**(g.)** $x^2 y' + x(x + 2)y = e^x$

**(h.)** $\cos^2 x \sin x dy + (y \cos^3 x - 1)dx = 0$

**(i.)** $\dfrac{dr}{d\theta} + r \sec\theta = \cos\theta$

**(j.)** $L\dfrac{di}{dt} + Ri = E$ where $L, R, E$ are nonzero constants and $i(0) = i_o$

**PP 8** (Bernoulli's Equation). If the DEqn has form $\frac{dy}{dx} + P(x)y = f(x)y^n$ for some real $n$ then it is called a Bernoulli Equation. These can be solved by a $w = y^{1-n}$ substitution, we assume $n \neq 0, 1$. Solve the following:

**(a.)** $\dfrac{dy}{dx} - y = e^x y^2$

**(b.)** $x\dfrac{dy}{dx} - (1 + x)y = xy^2$

**(c.)** $3(1 + x^2)\dfrac{dy}{dx} = 2xy(y^3 - 1)$

**PP 9** (Ricatti's Equation). If $\frac{dy}{dx} = P(x) + Q(x)y + R(x)y^2$ then the given DEqn is a Ricatti Equation. If $y_1$ is a known solution then the substitution $v = y_1 + u$ turns the problem into a Bernoulli Equation with $n = 2$. Given the Ricatti Equation below with known solution $y_1$, solve it. Or, if no $y_1$ is given then figure one out then solve it.

**(a.)** $\dfrac{dy}{dx} = 1 - x - y + xy^2$, $y_1 = 1$

**(b.)** $\dfrac{dy}{dx} = 2x^2 + y/x - 2y^2$, $y_1 = x$

**(c.)** $\dfrac{dy}{dx} = \sec^2 x - (\tan x)y + y^2$, $y_1 = \tan x$

**(d.)** $\dfrac{dy}{dx} = 9 + 6y + y^2$

**PP 10** (Clauiraut Equation) Let $f$ be a smooth function. The differential equation $y = xy' + f(y')$ is known as a Clairaut Equation. Show that $y = cx + f(c)$ serves as a solution to Clairaut Equation for any $c \in \mathbb{R}$. Furthermore, show

$$x = -f'(t), \qquad \& \qquad y = f(t) - tf'(t)$$

give a parametric solution to Clairaut Equation. If $f''(t) \neq 0$ then the parametric solution describes a solution not found in the linear family and as such it is known as the **singular solution**. Solve the Clairaut Equations below by finding both their linear solutions and the singular solution.

**(a.)** $y = xy' + (y')^{-2}$

**(b.)** $y = (x+4)y' + (y')^2$

**(c.)** $y - xy' = \ln y'$

**PP 11** Solve $2xy\dfrac{dy}{dx} + 2y^3 = 3x - 6$ by making a substitution of $v = y^2$.

**PP 12** Solve $y'' = 2x(y')^2$ by making a $v = y'$ substitution.

**PP 13** Solve $\dfrac{dy}{dx} = e^{x-y}\cosh x$

**PP 14** Solve $\dfrac{dy}{dx} = \dfrac{y^2+4y+5}{x^2-3x-4}$

**PP 15** Solve $(y + \sin^{-1}(x))dx + \left(x + \dfrac{1}{1+y^2}\right)dy = 0$

**PP 16** Solve $\dfrac{dy}{dx} = \dfrac{y^2+2xy}{x^2}$. Hint: write as $\dfrac{dy}{dx} = F(y/x)$.

**PP 17** Find the explicit solution of $\dfrac{dy}{dx} = \dfrac{e^x}{y}$ for which $y(0) = -2$.

**PP 18** Find the explicit general solution of $\dfrac{dy}{dx} + (\tan x)y = \sec x$.

**PP 19** Find a continuous solution of $\frac{dy}{dx} = |x-3|$ which contains the origin.

**PP 20** Find a continuous solution of $\frac{dy}{dx} = \sqrt{(x-3)^2} - \sqrt{(x-4)^2}$ which contains the origin.

**PP 21** Find the explicit solution of $\frac{dy}{dx} - \frac{3y}{x} = \sqrt{x}$.

**PP 22** Note that $\mu = 1/(yx^2)$ is an integrating factor for the following differential equation:

$(3x^2y+y^2)dx+(x^2y^2-xy)dy = 0,$

Find the general solution to the differential equation. If there are any exceptional solutions be sure to point them out.

**PP 23** Find the solution of $(2 + ye^{xy})\,dx + (y + xe^{xy})\,dy = 0$ through $(x_o, y_o)$. Hint: this is an exact DEqn thus you can use the theorem involving line integrals to build the desired solution

**PP 24** Solve $(2y\sec^2(x^2y))dx + (x\sec^2(x^2y) + \frac{1}{x}e^y)dy = 0$.
Hint: $\mu = x$ is an integrating factor for this inexact equation.

**PP 25** Solve $\frac{dy}{dx} = \frac{y^2+2xy}{x^2}$.
Hint: write as $\frac{dy}{dx} = F(y/x))$

**PP 26** Solve

$$\frac{dy}{dx} = y(xy^3 - 1).$$

**PP 27** Solve the following by making a subsitution which replaces $x$ and $y$ with polar coordinates $r$ and $\theta$. Please give your answer in terms of $r, \theta$.

$$[2x(x^2 + y^2) + y]dx + [2y(x^2 + y^2) - x]dy = 0.$$

**PP 28** Find the implicit solution of:

$$\left(1 + 2xy^2 - \frac{1}{x^2+4}\right) dx + \left(2y + 2x^2y - \frac{1}{1-y^2}\right) dy = 0.$$

**PP 29** Find the potential energy function for the conservative vector field:

$$\vec{F}(x,y) = \left\langle 1 + 2xy^2 - \frac{1}{x^2+4}, 2y + 2x^2y - \frac{1}{1-y^2} \right\rangle$$

**PP 30** Consider the differential equation $Pdx + Qdy = 0$ where $P_y = Q_x$ on a simply connected region $U \subseteq \mathbb{R}^2$. Use Calculus III to prove there exists $F : U \to \mathbb{R}$ for which $dF = Pdx + Qdy$ and explain why $F(x,y) = C$ serves to solve $Pdx + Qdy = 0$.

**PP 31** Consider the differential equation $Pdx + Qdy = 0$ where $P_y \neq Q_x$ on a simply connected region $U \subseteq \mathbb{R}^2$. Use Calculus III to prove there cannot exist $F : U \to \mathbb{R}$ for which $dF = Pdx + Qdy$.

**PP 32** Consider $\omega = \dfrac{-ydx + xdy}{x^2 + y^2}$.

**(a.)** Calculate $\int_C \omega$ where $C$ is the CCW unit-circle.

**(b.)** We say $\omega$ is closed if $d\omega = 0$. Show $\omega$ is closed.

**(c.)** We say $\omega$ is exact if there exists $F$ for which $dF = \omega$ on $U \subseteq \mathbb{R}^2$. Is $\omega$ exact on $\mathbb{R}^2$ ?

**(d.)** On which simply connected subsets of $\mathbb{R}^2$ is $\omega$ exact ?

**PP 33** The wedge product follows the usual rules of algebra except it satisfies $dx \wedge dx = 0$ and $dx \wedge dy = -dy \wedge dx$ etc. Calculate $(a_1dx + a_2dy + a_3dz) \wedge (b_1dx + b_2dy + b_3dz)$ and comment on the meaning of the constants $c_1, c_2, c_3$ with

$$(a_1dx + a_2dy + a_3dz) \wedge (b_1dx + b_2dy + b_3dz) = c_1dy \wedge dz + c_2dz \wedge dx + c_3dx \wedge dy$$

can you recognize $\langle c_1, c_2, c_3 \rangle$ as it relates to $\langle a_1, a_2, a_3 \rangle$ and $\langle b_1, b_2, b_3 \rangle$ ?

**PP 34** The exterior derivative of a one-form $\omega = Adx + Bdy + Cdz$ is given by $d\omega = dA \wedge dx + dB \wedge dy + dC \wedge dz$ where $dA = (\partial_x A)dx + (\partial_y A)dy + (\partial_z A)dz$ is the total differential you know and love from Calculus III. Let $f$ be a smooth function of $x, y, z$. Show that $d(df) = 0$. To which identity of vector calculus does your calculation correspond ?

**PP 35** A differential equation $Mdx + Ndy = 0$ is exact if there exists $F$ for which $dF = Mdx + Ndy$. Since $d(dF) = 0$ is an identity of the exterior calculus we can check on the exactness of a given differential equation in Pfaffian form by taking its exterior derivative. Determine if the differential equations below are exact by taking the exterior derivative of the differential equation:

**(a.)** $ydx + x^2dy = 0$

**(b.)** $y\sin(xy)dx + x\sin(xy)dy = 0$

**(c.)** $-x^2dy + y^2dx = 0$

**PP 36** A level surface $F(x, y, z) = 0$ has gradient vector field $\nabla F = \langle F_x, F_y, F_z \rangle$ which serves as a normal to the tangent plane of the given surface. Since $\omega_{\nabla F} = dF$ we need that $d\omega_{\nabla F} = d(dF) = 0$. Determine if the vector fields below are normal vector fields to some level surface in $\mathbb{R}^3$:

**(a.)** $\langle x, y, z \rangle$

**(b.)** $\langle y, 1, x \rangle$

**(c.)** $\langle xy^2 + z, x^2y, -x \rangle$

**PP 37** Recall the vector field $\vec{F} = \langle a, b, c \rangle$ corresponds to the **work form** $\omega_{\vec{F}} = adx + bdy + cdz$. Suppose $\vec{F} = \langle yz, x, 3, z^3 \rangle$ then write the formula for $\omega_{\vec{F}}$ and calculate $d\omega_{\vec{F}}$. Is $\vec{F}$ conservative on $\mathbb{R}^3$ ?

**PP 38** Recall the vector field $\vec{F} = \langle a, b, c \rangle$ corresponds to the **flux form** $\Phi_{\vec{F}} = ady \wedge dz + bdz \wedge dx + cdx \wedge dy$. Suppose $\vec{F} = \langle x + y, y + z, z + x \rangle$ then write the formula for $\Phi_{\vec{F}}$ and calculate $d\Phi_{\vec{F}}$. If we were to calculate the flux of $\vec{F}$ through the unit-sphere then what would the result be ?

**PP 39** (Orthogonal Trajectories) Find the orthogonal trajectories to the curve or family of curves described below:

**(a.)** $y = (x - c_1)^2$

**(b.)** $y = e^{c_1 x}$

**(c.)** $y = \frac{1+c_1x}{1-c_1x}$

**(d.)** $\sinh y = c_1 x$

**(e.)** $y^2 - x^2 = c_1 x^3$

**PP 40** (Polar Curves) Consider a curve with polar equation $r = f(\theta)$. Let $\Psi$ be the counterclockwise angle swept from the radial line to the tangent line along the curve. Show that $r\frac{d\theta}{dr} = \tan\Psi$. Then show that two polar curves are orthogonal if and only if $\tan\Psi_1 \tan\Psi_2 = -1$ at a point of intersection between curve $C_1$ and $C_2$. Use $r = f_1(\theta)$ for curve $C_1$ whereas $r = f_2(\theta)$ for $C_2$.

**PP 41** (Orthogonal Trajectories to Polar Curves) Find the orthogonal trajectory for the curves described below (please use polar coordinates to formulate the answer)

**(a.)** $r = c_1(1 + \cos\theta)$

**(b.)** $r = \frac{c_1}{1+\cos\theta}$

**(c.)** $r = c_1 e^{\theta}$

**PP 42** (Isogonal Families) A family of curves which intersects a given family of curves at an angle $\alpha \neq \pi/2$ are said to be **isogonal trajectories** of each other. If $\frac{dy}{dx} = f(x,y)$ describes a given family of curves then show its isogonal family are solutions of

$$\frac{dy}{dx} = \frac{f(x,y) \pm \tan\alpha}{1 \mp f(x,y)\tan\alpha}.$$

Then, find the isogonal family to $y = c_1 x$ at angle $\alpha = 30^o$.

**PP 43** Find a Cartesian equation which is parametrized by the solution of the following system of differential equations:

$$\frac{dx}{dt} = -y \qquad \& \qquad \frac{dy}{dt} = 2x.$$

**PP 44** An integral curve to a vector field $\vec{F} = \langle P, Q\rangle$ can be described parametrically as a path $t \mapsto \vec{\gamma}(t) = (x(t), y(t))$ for which $\vec{F}(\vec{\gamma}(t)) = \frac{d\vec{\gamma}}{dt}$. That is, $\langle P, Q\rangle = \langle dx/dt, dy/dt\rangle$. Parametrically we need to solve $\frac{dx}{dt} = P$ and $\frac{dy}{dt} = Q$. However, if we are only interested in describing the integral curve in Cartesian coordinates then we can eliminate $t$ via the calculus

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Q}{P}$$

thus finding an integral curve for a given vector field which depends only on $x, y$ is as simple as solving the above first order ODEqn.

**(a.)** Find integral curves to the vector field $\vec{F}(x,y) = \langle x^2, y^3\rangle$.

**(b.)** Consider the vector field $\vec{F}(x,y) = \left\langle \frac{y}{(x-1)^2+y^2}, \frac{1-x}{(x-1)^2+y^2} \right\rangle$. Find the the level curve which serves as an integral curve for $\vec{F}$ through $P_o = (x_o, y_o) \neq (1,0)$.

**(c.)** Find the integral curves of the vector field $\vec{F} = \langle 1, e^{x^3} - 2y/x\rangle$. Please leave your answer explicitly in terms of $y$ as a function of $x$.

**PP 45** Suppose $F_f = -\alpha v^3$ is the net-force acting on a mass $m$ in one-dimensional motion where the coordinate is denoted $x$ (here $\alpha$ is an appropriate dimensional constant). Suppose $v = v_o$ and $x = x_o$ when $t = 0$ and calculate:

(a) velocity as function of time $t$

(b) velocity as a function of position $x$

**PP 46** Let $b$ be a positive constant. If a friction force of $F_f = -bv^4$ is applied to a mass $m$ with initial position $x_o$ and initial velocity $v_o$ then find the velocity as a function of

**(a.)** time $t$,

**(b.)** position $x$.

**PP 47** A net-force of $F = \alpha t - kv$ is placed on a mass $m$ where $k, \alpha$ are constant. Find the velocity as an **explicit** function of time $t$. Assume that the particle undergoes one-dimensional motion with velocity $v_o$ when $t = 0$.

**PP 48** A large tank initially has 20 lbs of Koolaid mix added to 1000 *gallons* of water. Studies have shown that children will only drink the Koolaid when there is at least 0.005 *lbs* per gallon of water. You work for an incredibly lazy camp director who insists on adding the Koolaid to the 1000 *gallon* mixing tank only when the kids finally start rejecting the sadly weak Koolaid. Given that 25 *gallons* of pure water are added to the tank every day to make up for the 25 *gallons* of Koolaid drunk by the kids then when will you have to ask the camp director to add more mix ?

**PP 49** A tank initially contains 100 gallons of water with 10lb of lemon drink mix. Then at $t = 0$ fresh water is added to the tank at 3 gallons per minute and at the same time 3 gallons are drained per minute from the tank. Assume the tank is well-mixed during this process. Find the lb's of lemon drink mix as a function of time. If you like your drink with a concentration of 1lb per 20 gallons then at what time should you drink from the drain?

**PP 50** Suppose a mixing tank is well-stirred and contains $y$ kilograms of salt at time $t$. Suppose pure water flows into the tank at a rate of 4 Liters per minute and salty water flows out at a rate of 2 Liters per minute. If the tank has 300 Liters of liquid and $300kg$ at time zero then write (but do not solve) the differential equation which describes the change in $y$.

**PP 51** The radioactive lead isotope Pb-209 has a half-life of 3.3 hours. If 1 kilogram is initially present then how long will it take for there to be only 0.1 kilograms of radioactive lead remaining ?

**PP 52** Show that the half-life of a radiactive substance is given by

$$t = \frac{(t_2 - t_1)\ln 2}{\ln(A_1/A_2)}$$

where $A_1 = A(t_1)$ and $A_2 = A(t_2)$ where $t_1 < t_2$.

**PP 53** When a vertical beam of light passes through a transparent substance the rate at which its intensity $I$ decreases is proportional to $I(y)$ where $y$ represents the thickness of the medium in feet. In clear calm seawater the intensity 3 feet below the surface is 1/4 the initial intensity of the incident beam at the surface. What is the intensity of the beam 15 feet below the surface ?

**PP 54** A thermometer is removed from a room where the air temperature is $70^o$ F to the outside where the temperature is $10^o$ F. After 0.5 minutes the thermometer reads $50^o$ F. What is the reading at $t = 1.0$ minutes ? How long will it take for the thermometer to reach $15^o$ F ?

**PP 55** When a resistor $R$ and inductor $L$ are in series with a voltage source $\mathcal{E}$ then circuit analysis yields the differential equation:

$$L\frac{di}{dt} + Ri = \mathcal{E}$$

where $i$ is the current flowing in the circuit. Given $\mathcal{E}(t) = V_o \sin \omega t$ and $i(0) = i_o$ find the current as a function of time $t$.

**PP 56** Angular momentum of a body moving in some plane is given by $L = mr^2\frac{d\theta}{dt}$ where $r, \theta$ serve as polar coordinates in the plane of motion. Assume the coordinates of the body are $(r_1, \theta_1)$ at $t = t_1$ and $(r_2, \theta_2)$ at $t = t_2$ where $t_1 < t_2$. **If $L$ is constant then show that the area swept out by $r$ is $A = L(t_2 - t_1)/2m$.** When the sun is taken to be at the origin and $m$ represents a planet's mass then this proves Kepler's second law of planetary motion: the radius vector joining the sun sweeps out equal areas for equal intervals of time. **Bonus: prove $L$ is constant in the context of the sun-planet system, you may assume $M_{sun} >> M_{planet}$. See Physics 231 for further relevant definitions.**

**PP 57** Find the velocity of a mass $m$ which is launched vertically with velocity $v_o$ from a planet with mass $M$ and radius $R$. Recall that the gravitational force is given by:

$$F = -\frac{GmM}{(R+y)^2}$$

if we assume the motion is directly vertical and $y$ is the altitude of $m$. You may find the velocity as a function of $y$.

**PP 58** Suppose a rocket car has an initial speed of $v_o$ as it hurtles across a speedway in a remote desert. Suppose the driver opens a parachute which developes a retarding force proportional to the cube of the velocity; $F_f = -kv^3$. Find the velocity as:

(a.) a function of time,

(b.) a function of position $x$ taking $x_o$ as the initial position

**PP 59** A ball is thrown verically and its motion is governed by the force of gravity $-mg$ and a friction force $F_f = -cv$ where $v = \dot{y}$ and $c$ is a constant which is based on the size of the ball. Note: $F_f > 0$ when $v < 0$ and $F_f < 0$ when $v > 0$ hence the friction force acts opposite of the motion.

(a) find the velocity as a function of time and the initial velocity $v_o$

(b) find the position $y$ as a function of time and the initial position $y_o$

(c) if $c = 0$ then what is the maximum height reached by the ball? Does it depend on the $m$?

(d) if $c \neq 0$ then what is the maximum height reached by the ball? Does it depend on the $m$?

**PP 60** Suppose a $10kg$ block is pushed across a surface by a constant force of $10N$. As the block moves it gathers a gummy substance which results in a friction force of magnitude $F_o e^{kt}$ where $F_o = 1N$ and $k = 1/s$. Find the position $x$ of the block as a function of time $t$. Assume that at time $t = 0$ the block is at $x = 0$. (feel free to use technology to do the numerical aspects here)

**PP 61** A chain is coiled on the ground. One end is then lifted with constant force. Find the velocity.

**PP 62** Suppose the $RL$-circuit has a voltage source which varies with time according to $\mathcal{E}(t) = V_o \cos(t)$. Find the current as a function of time and the initial current $I_o$. *hint: this is like an example in the notes, just replace the constant $\mathcal{E}$ with the sinuisoidal source $\mathcal{E}(t) = V_o \cos(t)$*

**PP 63** Find a continuous function $P$ such that $P(x) = a + \int_0^x t^2 P(t)\, dt$.

**PP 64** Suppose $f$ is a function such that $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. Show that either $f$ is the function which is identically zero on $\mathbb{R}$ or$f$ is an exponential function.

**PP 65** Let $\star$ be the DEqn $y^2 \sin(x)dx + yf(x)dy = 0$. Find all functions $f$ such that $\star$ is an exact DEqn.

**PP 66** Explicitly solve $\dfrac{dy}{dx} = \dfrac{2x + 3x^2}{2y}$ given that the point $(1, -2)$ is on the solution.

**PP 67** Solve $\dfrac{dy}{dx} - \dfrac{3}{x}y = x^3 e^x$

**PP 68** Solve $\left(\frac{1}{x+1} + y\cos(xy)\right) dx + \left(e^{-y} + x\cos(xy)\right) dy = 0$.

**PP 69** Find orthogonal trajectories of $\frac{dy}{dx} = \frac{-x}{y}$.

**PP 70** Find the velocity as a function of time $t$ given that $v = v_o$ when $t = t_o$ and $F_{net} = -\beta v^2$ for a mass $m$.

**PP 71** Solve $\frac{dP}{dt} = kP(P - C)$. Describe the possible solutions. You should find three disjoint types. Assume $k > 0$.

**PP 72** Consider the following four differential equations:

(I.) $\frac{dy}{dx} = 1/2$

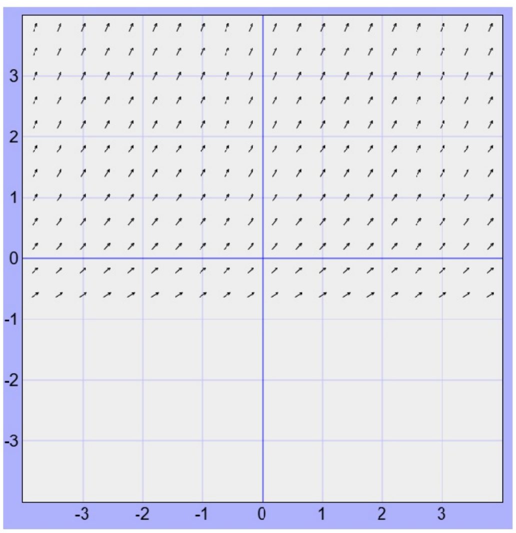
(II.) $\frac{dy}{dx} = \sqrt{y+1}$

(III.) $\frac{dy}{dx} = (y-2)^2(x-1)$

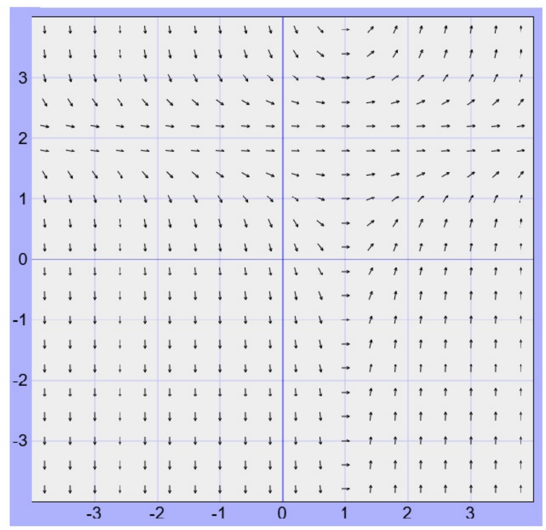
(IV.) $\frac{dy}{dx} = (x^2-1) * y^2$

I used https://aeb019.hosted.uark.edu/pplane.html to generate the following direction fields. Match A,B,C,D with the corresponding I,II,III,IV.

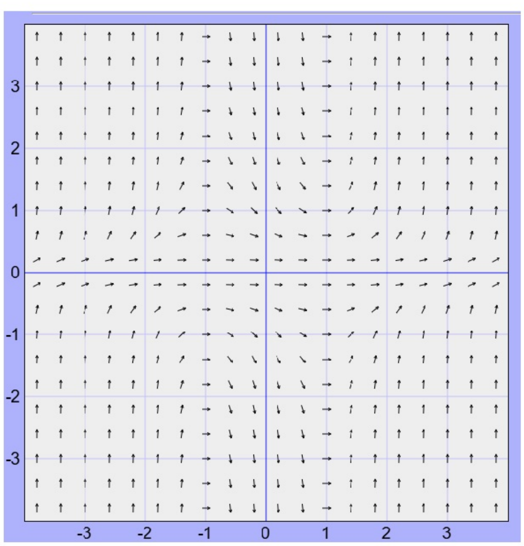
**(A)**



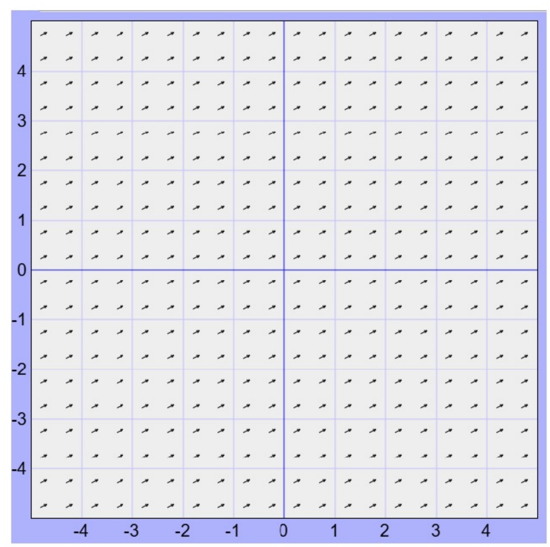

**(B)**




**(C)**




**(D)**




**Remark:** the pplane tool is an easy way to explore the behavior of a given differential equation without going to the trouble of solving it. To visualize $\frac{dy}{dx} = f(x,y)$ I set $dx/dt = 1$ and $dy/dt = f(x,y)$. I didn't click on any points for the above plots, but if you do it traces out solutions. This can also illustrate solutions to systems of two autonomous ODEs, we'll get to that a bit later in the course.