Name: (please print name here →) .

MATH 334:	MISSION 1: FIRST ORDER DEQNS [50PTS]

You write the solution neatly in the box provided and show work on your own, standard sized, non-fuzzy edged, paper. The work must be labeled with problem number and part as appropriate and if a problem is skipped it must be mentioned in the attached work. Neatness is part of the score. Each part is worth 1pt unless otherwise indicated. 5pts are for following format. Enjoy.

Problem 1 (Separation of Variables) Solve the differential equations below. If possible, find the explicit solution, otherwise find an implicit general solution.

(a.) $\frac{dy}{dx} = (x+1)^2$

(b.) $e^x \frac{dy}{dx} = 2x$

(c.) $\frac{dy}{dx} = \frac{y+1}{x}$

(d.) $x^2y^2dy = (y+1)dx$

(e.) $\sec x dy = x \cot y dx$

(f.) $\sec y \frac{dy}{dx} + \sin(x-y) = \sin(x+y)$

(g.) $(e^x + e^{-x})\frac{dw}{dx} = w^2$

Problem 2 (Initial Value Problems) Use separation of variables to solve the IVPs below:

(a.) $\frac{dx}{dy} = 4(x^2+1)$ with $x(\pi/4) = 1$.

(b.) $y' + 2y = 1$ with $y(0) = 5/2$.

Problem 3 (exact equations) If the DEqn below is exact then solve it, otherwise explain why the given DEqn is not exact.

(a.) $(2xz^2 - 3)dx + (2zx^2 + 4)dz = 0$

(b.) $\left(2y - \dfrac{1}{x} + \cos 3x\right)\dfrac{dy}{dx} + \dfrac{y}{x^2} - 4x^3 + 3y\sin 3x = 0$

(c.) $(\theta^3 + \beta^3)d\theta + 3\theta\beta^2 d\beta = 0$

(d.) $(e^y + 2xy\cosh x)y' + xy^2\sinh x + y^2\cosh x = 0$

(e.) $\left(\dfrac{1}{x} + \dfrac{1}{x^2} - \dfrac{t}{x^2+t^2}\right)dx + \left(te^t + \dfrac{x}{x^2+t^2}\right)dt = 0$

Problem 4 (exact equations with integrating factor) A general form of an integrating factor is suggested. Find the specific form I which serves as an integrating factor and solve the DEqn $Mdx + Ndy = 0$ by solving the exact equation $IMdx + INdy = 0$)

(a.) $y(x + y + 1)dx + (x + 2y)dy = 0$ given $I = e^{Ax}$

(b.) $y(4xy^5 + 3)dx - x(2xy^5 + 7)dy = 0$ given $I = x^A y^B$

Problem 5 (linear first order DEqn) Solve the linear first order ODEqn given below and state the interval on which the solution is defined. If given an initial value, then fit the given data to the explicit solution.

(a.) $y' + 3x^2y = x^2$

(b.) $x\dfrac{dy}{dx} + 2y = 3$

(c.) $x\dfrac{dy}{dx} + 4y = x^3 - x$

(d.) $\cos^2 x \sin x dy + (y\cos^3 x - 1)dx = 0$

(e.) $\dfrac{dr}{d\theta} + r\sec\theta = \cos\theta$

Problem 6 (Bernoulli's Equation). If the DEqn has form $\frac{dy}{dx} + P(x)y = f(x)y^n$ for some real n then it is called a Bernoulli Equation. These can be solved by a $w = y^{1-n}$ substitution, we assume $n \neq 0, 1$. Solve the following:

(a.) $\dfrac{dy}{dx} - y = e^x y^2$

(b.) $3(1+x^2)\dfrac{dy}{dx} = 2xy(y^3 - 1)$

Problem 7 Find a continuous solution of $\frac{dy}{dx} = \sqrt{(x-3)^2}$ which contains the origin.

Problem 8 Solve the following by making a subsitution which replaces x and y with polar coordinates r and θ. Please give your answer in terms of r, θ.

$$[2x(x^2+y^2)+y]dx + [2y(x^2+y^2)-x]dy = 0.$$

Problem 9 Solve $\dfrac{dy}{dx} = e^{x-y}\cosh x$

Problem 10 Solve $\dfrac{dy}{dx} = \dfrac{y^2+4y+5}{x^2-3x-4}$

Problem 11 Solve $(y + \sin^{-1}(x))dx + \left(x + \dfrac{1}{1+y^2}\right)dy = 0$

Problem 12 Find the explicit solution of $\dfrac{dy}{dx} = \dfrac{e^x}{y}$ for which $y(0) = -2$.

Problem 13 Find the implicit solution of:

$$\left(1 + 2xy^2 - \frac{1}{x^2+4}\right) dx + \left(2y + 2x^2 y - \frac{1}{1-y^2}\right) dy = 0.$$

Problem 14 A differential equation $Mdx + Ndy = 0$ is exact if there exists F for which $dF = Mdx + Ndy$. Since $d(dF) = 0$ is an identity of the exterior calculus we can check on the exactness of a given differential equation in Pfaffian form by taking its exterior derivative. Determine if the differential equations below are exact by taking the exterior derivative of the differential equation:

(a.) $y\sin(xy)dx + x\sin(xy)dy = 0$

(b.) $-x^2dy + y^2dx = 0$

Problem 15 (Orthogonal Trajectories) Find the orthogonal trajectories to the curve or family of curves described below:

(a.) $y = (x - c_1)^2$

(b.) $y^2 - x^2 = c_1 x^3$

Problem 16 (Isogonal Families) A family of curves which intersects a given family of curves at an angle $\alpha \neq \pi/2$ are said to be **isogonal trajectories** of each other. If $\frac{dy}{dx} = f(x,y)$ describes a given family of curves then show (4pts) its isogonal family are solutions of

$$\frac{dy}{dx} = \frac{f(x,y) \pm \tan\alpha}{1 \mp f(x,y)\tan\alpha}.$$

Then, find (1pt) the isogonal family to $y = c_1 x$ at angle $\alpha = 30^o$.

Problem 17 An integral curve to a vector field $\vec{F} = \langle P, Q \rangle$ can be described parametrically as a path $t \mapsto \vec{\gamma}(t) = (x(t), y(t))$ for which $\vec{F}(\vec{\gamma}(t)) = \frac{d\vec{\gamma}}{dt}$. That is, $\langle P, Q \rangle = \langle dx/dt, dy/dt \rangle$. Parametrically we need to solve $\frac{dx}{dt} = P$ and $\frac{dy}{dt} = Q$. However, if we are only interested in describing the integral curve in Cartesian coordinates then we can eliminate t via the calculus

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Q}{P}$$

thus finding an integral curve for a given vector field which depends only on x, y is as simple as solving the above first order ODEqn.

(a.) Consider the vector field $\vec{F}(x,y) = \left\langle \frac{y}{(x-1)^2+y^2}, \frac{1-x}{(x-1)^2+y^2} \right\rangle$. Find the the level curve which serves as an integral curve for $\vec{F}$ through $P_o = (x_o, y_o) \neq (1,0)$.

(b.) Find the integral curves of the vector field $\vec{F} = \langle 1, e^{x^3} - 2y/x \rangle$. Please leave your answer explicitly in terms of y as a function of x.

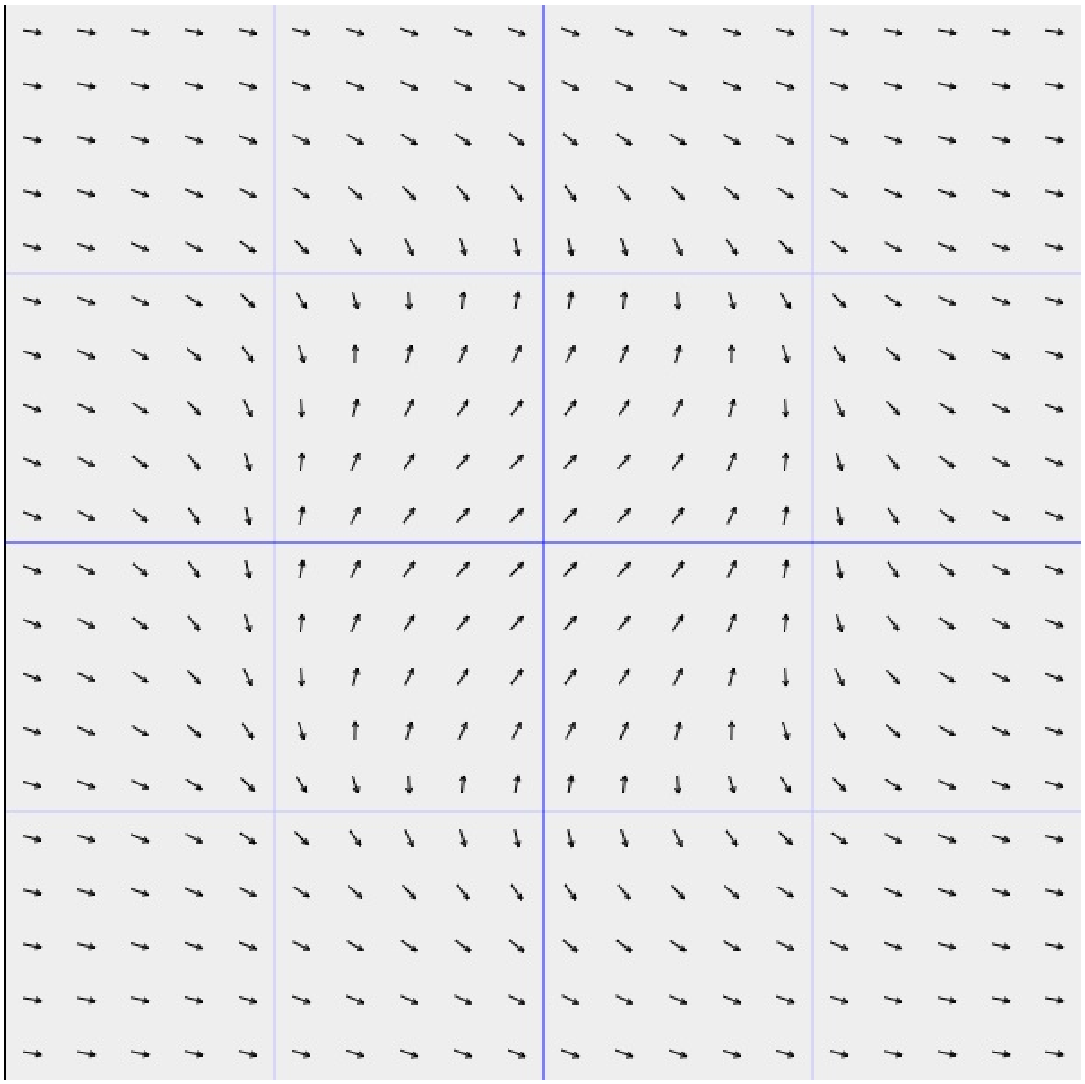
Problem 18 Let b be a positive constant. If a friction force of $F_f = -bv^4$ is applied to a mass m with initial position x_o and initial velocity v_o then find the velocity as a function of

(a.) time t,

(b.) position x.

Problem 19 A chain is coiled on the ground. One end is lifted with a constant force F. Find the velocity. Assume the chain uniform weight-density w per foot and use x for the length lifted in feet.

Problem 20 Plot solution curves for the direction field of $\frac{dy}{dx} = \frac{1}{1-x^2-y^2}$ given below and explain what is happening at the unit-circle for the solutions. Try it out in the pplane to check your hand-drawn answers here (maybe starting with a pencil lightly then tracing over with pen once you're sure would be wise here)



Remark: I used https://aeb019.hosted.uark.edu/pplane.html to generate the direction field above.