Name: (please print name here →) .

Math 334:	Mission 2: n-th Order DEqns [50pts]

You write the solution neatly in the box provided and show work on your own, standard sized, non-fuzzy edged, paper. The work must be labeled with problem number and part as appropriate and if a problem is skipped it must be mentioned in the attached work. Neatness is part of the score. There are more than 50pts that can be earned, but 50pts are in the Syllabus. Enjoy.

Problem 25 Determine if $\{\sin^2 x, \cos^2 x, 1\}$ is linearly independent on $\mathbb{R}$.

Problem 26 Suppose $y_1 = t^2e^t$ and $y_2 = te^t$ and $y_3 = e^t$. Calculate $W(y_1, y_2, y_3; t)$.

Problem 27 Let us define $L[y] = y''' + y' + xy$. Let $y_1 = \sin x$ and $y_2 = x$.

(a.) Calculate $L[y_1]$ and $L[y_2]$,

(b.) Solve $L[y] = 2x\sin x - x^2 - 1$,

(c.) Solve $L[y] = 4x^2 + 4 - 6x\sin x$.

I am not asking for the general solution in the problem above

Problem 28 (IVP) Solve $y'' + 2y' + y = 0$ where $y(0) = 1$ and $y'(0) = -3$ given $y' = dy/dx$.

Problem 29 (IVP) Solve $(D^2 - 2D + 2)[y] = 0$ given $y(\pi) = e^\pi$ and $y'(\pi) = 0$ and $D = d/dt$.

Problem 30 (Constant Coefficient Problems) Find the general solution for each DEqn below: assume the independent variable in each solution is denoted by x.

(a.) $y'' - y' - 11y = 0$

(b.) $4w'' + 20w' + 25w = 0$

(c.) $y'' - 8y' + 7y = 0$

(d.) $z'' + 10z' + 25z = 0$

(e.) $u'' + 7u = 0$

(f.) $y'' + 10y' + 41y = 0$

(g.) $y''' + 2y'' - 8y' = 0$

(h.) $u''' - 9u'' + 27u' - 27u = 0$

(i.) $y^{(4)} + 4y'' + 4y = 0$

(j.) $(D^4 - 36)[y] = 0$

(k.) $((D + 1)^2 + 36)^2[y] = 0$

Problem 31 Solve $y^{(4)} + 2y''' + 10y'' + 18y' + 9y = 0$ given that $y = \sin(3x)$ is a solution.

Problem 32 Use the method of annihilators to set-up (but do not determine explicitly) the particular solutions for:

(a.) $y'' - 2y' + y = x^2e^x$

(b.) $y'' + 16y = e^x\cos(4x)$

(c.) $y''' + y' = x^3 + e^x\cos(4x)$

(d.) $y'''+36y' = \cos^2(3x)+e^x\cosh(x)$

Problem 33 Consider $y'' - 6y' - 4y = 4\sin(3t) - t^2e^{3t} + \frac{1}{t}$. Can we solve this via the method of undetermined coefficients ? If so, suggest a form for the particular solution.

Problem 34 (nonhomogeneous problems) Find the general solution for each DEqn below:

(a.) $y'' - y = 1 - 11t$.

(b.) $y'' - 9y = t^2 + e^t + 1$.

(c.) $y'' + 3y' + 2y = t + 1$.

(d.) $y'' + y = \cos t + e^t$

Problem 35 Solve $z'' + z = 2e^{-x}$ given $z(0) = 0$ and $z'(0) = 0$.

Problem 36 Solve $y'' + 2y' + y = \dfrac{e^{-x}}{x+1}$.

Problem 37 Solve $y'' + 4y = \tan(2x)$.

Problem 38 Solve $y'' + 3y' + 2y = \sin(e^x)$

Problem 39 Solve the following cauchy euler problems. Give your solution as a real linear combination of the real-valued functions in the fundamental solution set.

(a.) $4x^2y'' + y = 0$

(b.) $x^2y'' - 3xy' + 5y = 0$

(c.) $2x^2y'' + 3xy' - y = 0$

(d.) $x^3y''' + 2x^2y'' - xy' + y = 0$

Problem 40 Find an integral solution for $x > 0$ to the Cauchy Euler problem $x^2y'' + xy' + 9y = g$ where g is a continuous function.

Problem 41 (based on Cook section 3.7) Suppose $T = D$ and $S = 3 - x^2D$. Solve

(a.) $ST[y] = 0$,

(b.) $TS[y] = 0$.

Problem 42 A spring has mass $m = 1$, coefficient of damping $\beta = 4$ and a spring constant $k = 5$. Find the general solution of Newton's Second Law.

Problem 43 Newton's Law for a retarded spring-mass system with external force f yield

$$m\ddot{x} + \beta\dot{x} + kx = f$$

Given $m = 2$, $b = 0$, $k = 32$ and $f = 68e^{-2t}\cos(4t)$ find the equation of motion given the system has initial conditions $x(0) = \dot{x}(0) = 0$.

Problem 44 Consider Newton's Second Law for mass-spring system under a sinusoidal force:

$$\ddot{x} + \omega^2 x = F_o \cos\gamma t$$

given $x(0) = \dot{x}(0) = 0$. Here $F_o, \omega\gamma$ are nonzero constants.

(a.) Find $x(t)$ given that $\gamma \neq \omega$

(b.) Calculate $x_r(t) = \lim_{\gamma\to\omega} x(t)$

(c.) Contrast the motion of $x(t)$ and $x_r(t)$ as $t \to \infty$

Problem 45 Kirchoff's Voltage Law for an RLC-circuit with voltage source $\mathcal{E}$ is given by

$$L\frac{dI}{dt} + RI + \frac{1}{C}Q = \mathcal{E}$$

Since $I = \frac{dQ}{dt}$ we find $L\ddot{Q} + R\dot{Q} + Q/C = \mathcal{E}$. Given that $L = 1$ and $R = 2$ and $C = 0.25$ and $\mathcal{E} = 50\cos t$ find the charge Q as a function of time t given the initial charge and current are both zero for $t = 0$.

Problem 46 If we study the motion of an spring

$$m\ddot{x} + \beta\dot{x} + kx = F$$

such that $\beta^2 - 4mk < 0$ then it is known as **underdamped motion**. If the external force $F = F_o\cos(\gamma t)$ then we find the motion is dominated by the particular solution as $t \to \infty$. Let $\omega = \frac{\sqrt{4mk-\beta^2}}{2m}$, then the homogeneous solution $x_h(t) =$

$e^{\frac{-\beta t}{2m}}(c_1\cos(\omega t) + c_2\sin(\omega t)) \to 0$ as $t \to \infty$. It can be shown that the particular solution of such a system is given by

$$x_p = \frac{F_o \sin(\gamma t + \phi)}{\sqrt{(k - m\gamma^2)^2 + \beta^2\gamma^2}}$$

where ϕ is a constant. Find the frequency γ which maximizes the magnitude of x_p in the following cases:

(a.) $m = 1/2$ and $k = 19$ and $\beta = 1$

(b.) $m = 1$ and $k = 2$ and $\beta = \sqrt{6}$.

Problem 47 Solve the integral $\int(x^3 + 2x)e^x\,dx = y$ by solving $\frac{dy}{dx} = (x^2 + 2x)e^x$ via the method of undetermined coefficients

Problem 48 Find the general solution of

(a.) $y'' + y - 2y = x^2 + 3x$

(b.) $y'' + y - 2y = x\cosh x$

(c.) $y'' + y - 2y = 2(x^2 + 3x) + 10\cosh(x)$