I will select some subset of these problems to collect. The more you work, the more you know. The ordering of topics in these problems is rather lumpy, I've more or less cut and pasted multiple old homeworks and quizzes one after the other.

**PP 73** Solve $y'' - y' - 11y = 0$ where $y' = dy/dx$.

**PP 74** Solve $4w'' + 20w' + 25w = 0$ where $w' = dw/dx$.

**PP 75** Solve $y'' + 2y' + y = 0$ where $y(0) = 1$ and $y'(0) = -3$ given $y' = dy/dx$.

**PP 76** Suppose $y_1 = te^{2t}$ and $y_2 = e^{2t}$. Determine if $y_1$ and $y_2$ are linearly dependent on $(0, 1)$.

**PP 77** Solve $y'' - 8y' + 7y = 0$ where $y = y(t)$.

**PP 78** Solve $z'' + 10z' + 25z = 0$ where $z' = dz/dx$

**PP 79** Solve $u'' + 7u = 0$ given $t$ is the independent variable.

**PP 80** Solve $y'' + 10y' + 41y = 0$ given $y' = dy/dx$.

**PP 81** Solve $y'' - 2y' + 2y = 0$ given $y(\pi) = e^{\pi}$ and $y'(\pi) = 0$. Use independent variable $x$.

**PP 82** Consider $y''-6y'-4y = 4\sin(3t)-t^2e^{3t}+\frac{1}{t}$. Can we solve this via the method of undetermined coefficients ? If so, suggest a form for the particular solution.

**PP 83** Consider $y''-2y'+3y = \cosh t$. Can we solve this via the method of undetermined coefficients ? If so, suggest a form for the particular solution.

**PP 84** Find the general solution to $y'' - y = 1 - 11t$.

**PP 85** Solve $z'' + z = 2e^{-x}$ given $z(0) = 0$ and $z'(0) = 0$.

**PP 86** Determine if $\{\sin^2 x, \cos^2 x, 1\}$ is linearly independent on $\mathbb{R}$.

**PP 87** Show $\{x, x^2x^3, x^4\}$ is linearly independent on $\mathbb{R}$.

**PP 88** Let us define $L[y] = y''' + y' + xy$. Let $y_1 = \sin x$ and $y_2 = x$.

**(a.)** Calculate $L[y_1]$ and $L[y_2]$,

**(b.)** Solve $L[y] = 2x\sin x - x^2 - 1$,

**(c.)** Solve $L[y] = 4x^2 + 4 - 6x\sin x$.

*I am not asking for the general solution in the problem above*

**PP 89** Solve $y''' + 2y'' - 8y' = 0$ given $y' = dy/dt$.

**PP 90** Solve $u''' - 9u'' + 27u' - 27u = 0$ given $u = u(x)$.

**PP 91** Solve $y^{(4)} + 4y'' + 4y = 0$ given $y = y(x)$.

**PP 92** Solve $y^{(4)} + 2y''' + 10y'' + 18y' + 9y = 0$ given that $y = \sin(3x)$ is a solution.

**PP 93** Let $D = d/dx$. Solve

$$(D+1)^2(D-6)^3(D+5)(D^2+1)(D^2+4)[y] = 0.$$

**PP 94** Completely factor the following polynomials over $\mathbb{R}$. Place any irreducible quadratic factors in the completed-square format $(x-\alpha)^2 + \beta^2$.

**(a.)** $x^2 + 6x + 20$

**(b.)** $x^4 + 5x^2 - 6$

**(c.)** $x^4 - 256$

**(d.)** $f(x) = -20 - 36x - 15x^2 + 5x^3 + 5x^4 + x^5$ given that $f(-1) = 0$ and $f(-2+i) = 0$

**PP 95** Find the general solutions of the DEqns given below.

**(a.)** $y'' + 6y' + 20y = 0$

**(b.)** $(D^4 + 5D^2 - 6)[y] = 0$

**(c.)** $y^{(4)} - 256y = 0$

**(d.)** $-20y - 36y' - 15y'' + 5y''' + 5y^{(4)} + y^{(5)} = 0$
given that $y_1 = e^{-x}$ and $y_2 = e^{-2x}\cos(x)$ are solutions.

**PP 96** Solve the following ODE,

$$(D^2 + 6D + 13)(D^2 - 9)(D^2 + 1)(D^2 + 4D + 3)[y] = 0.$$

**PP 97** Find minimal annihilators for each of the functions below:

**(a.)** $f_1(x) = x^2 e^x$

**(b.)** $f_2(x) = e^x \cos(4x)$

**(c.)** $f_3(x) = x^3 + e^x \cos(4x)$

**(d.)** $f_4(x) = \cos^2(3x) + e^x \cosh(x)$

**PP 98** Set-up, but do not determine explicitly, the particular solutions for:

**(a.)** $y'' - 2y' + y = x^2 e^x$

**(b.)** $y'' + 16y = e^x \cos(4x)$

**(c.)** $y''' + y' = x^3 + e^x \cos(4x)$

**(d.)** $y''' + 36y' = \cos^2(3x) + e^x \cosh(x)$

**PP 99** Solve $y'' + 3y' + 2y = x + e^{-x} + e^{3x}$.

**PP 100** Solve $y'' + 3y' + 2y = e^{-2t}\cos(t)$.

**PP 101** Solve $y'' + 3y' + 2y = 20(t + e^{-t} + e^{3t}) + 2e^{-2t}\cos(t)$ given that $y(0) = 0$ and $y'(0) = 1$.

**PP 102** Solve $y'' + 2y' + y = \frac{e^{-x}}{x+1}$.

**PP 103** Solve $y'' + y = \tan^2(x)$

**PP 104** Find integral solutions for $y''' + 16y' = f$. (you need to use variation of parameters, I would explicitly calculate the determinants of $S_1, S_2, S_3$ as I discuss in the notes)

**PP 105** Solve $x^2y'' - (x^2+2x)y' + (x+2)y = x^3$. Note $y_1 = x$ is a fundamental solution of the DEqn. *Hint: find the 2nd. fundamental soln. and then use variation of parameters to find* $y_p$...

**PP 106** Solve the following cauchy euler problems. Give your solution as a real linear combination of the real-value functions in the fundamental solution set.

**(a.)** $4x^2y'' + y = 0$

**(b.)** $x^2y'' - 3xy' + 5y = 0$

**(c.)** $2x^2y'' + 3xy' - y = 0$

**(d.)** $x^3y''' + 2x^2y'' - xy' + y = 0$

**(e.)** $x^2y'' + 5xy' + 4y = 0$ with $y(1) = 2$ and $y'(1) = -3$

**PP 107** Derive a formula to rewrite $x^4D^4$ as a polynomial in $xD$. Use the result to solve $x^4D^4[y] = 0$. Please use my notes for formulas for $x^3D^3$ and $x^2D^2$, also, use Leibniz product rule for best results.

**PP 108** Suppose a mass of 1kg is attached to a spring with stiffness 5 Newtons per meter. Then the spring and mass are immersed in an oil with viscosity producing a velocity-dependent friction force with coefficient $\beta = 4kg/s$. If a force $F(t) = 10\cos(t)$ (in Newtons and seconds) is used to drive the system then what is the resulting equation of motion? Assume that $x(0) = 0$ and $v(0) = 1$. What anglular frequency $\gamma$ would make the force $10\cos(\gamma t)$ give a particular solution of maximum amplitude?

**PP 109** Suppose an RLC-circuit is assembled with $R = 11\Omega$, $L = 1H$ and $C = 0.1F$. If a half-decaying voltage source of $\mathcal{E}(t) = 10e^{-t} + \cos(t)$ is attached to the circuit then what is the resulting current as a function of time. Assume a switch closes at $t = 0$ connecting the voltage source to the circuit. This means $I(0) = 0$ and $Q(0) = 0$.

**PP 110** Let $f$ and $g$ be functions which are twice continuously differentiable on an interval $I$ for which $W(f, g; x) \neq 0$ for each $x \in I$. Show that

$$det \begin{bmatrix} y & y' & y'' \\ f & f' & f'' \\ g & g' & g'' \end{bmatrix} = 0$$

is a second order, linear, homogeneous differential equation with fundamental solutions $y_1 = f$ and $y_2 = g$. Then, use this result to construct a differential equation which has solutions $e^x$ and $e^{1/x}$, include the interval on which these are the solutions.

**PP 111** Show that the Cauchy-Euler problem

$$a_o \frac{d^n y}{x^n} + a_1 \frac{d^{n-1} y}{x^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0$$

problem changes to a constant coefficient problem if we make the substitution $x = e^t$. Use this result to derive the solutions of the Cauchy-Euler problem for which we find $R = 1$ three times, or $R = 1 + 2i$ twice.

**PP 112** Novel methods of integration.

**(a.)** Solve $\int x^3 e^x \, dx$ by solving $\frac{dy}{dx} = x^3 e^x$ using the method of undetermined coefficients.

**(b.)** Solve $\int e^x \cos(2x) \, dx$ by studying the integral of $\int e^{(1+2i)x} \, dx$. *Hint: we know* $\frac{d}{dx} e^{\lambda x} = \lambda e^{\lambda x}$ *even for the case* $\lambda = 1 + 2i$.

**PP 113** Let $y_1$ and $y_2$ form the fundamental solution set of the second order linear differential equation

$$a_o y'' + a_1 y' + a_2 y = 0$$

on an interval $I$. Show that between any two successive zeros of $y_1$ there is exactly one zero of $y_2$.

**PP 114** (Ritger & Rose section 5-4 problem 1a-d) find the general solution of:

**(a.)** $y'' = 0$

**(b.)** $y'' - 2y' = 0$

**(c.)** $y'' - a^2 y = 0$

**(d.)** $y'' + a^2 y = 0$

**PP 115** (Ritger & Rose section 5-4 problem 3) Suppose $ay'' + by' + cy = 0$ has distinct real characteristic values of $\lambda_\pm = A \pm B$ and hence a general solution $y = c_1 e^{\lambda_+ x} + c_2 e^{\lambda_- x}$. Show that the general solution can be rewritten as

$$y = e^{Ax}(b_1 \cosh(Bx) + b_2 \sinh(Bx)).$$

**PP 116** (Ritger & Rose section 5-5 problems 1,2,3 and 8)

**(1.)** $y'' + 3y' - 5y = 4e^{2x} + 6e^{-3x}$

**(2.)** $y'' + 3y' + 5y = 2\sin(3x)$

**(3.)** $y'' + 9y = 4\cos(3x)$

**(8.)** $y'' - 3y' = 2x^2 + 3e^x$

**PP 117** (introduction to theory of adjoints, from page 95 of Boyce and DiPrima's 3rd Ed.) If

$$p(x)y'' + q(x)y' + r(x)y = 0$$

can be expressed as $[p(x)y']' + [f(x)y]' = 0$ then it is said to be **exact**. Omit $x$-dependence in $p, q, r, \mu$ for brevity, if $py'' + qy' + ry = 0$ is not exact then it is possible to make it exact with multiplication by the appropriate integrating factor $\mu$. **Show** that for $\mu$ to accomplish its stated task it must itself be the solution of the so-called **adjoint equation**

$$p\mu'' + (2p' - q)\mu' + (p'' - q' + r)\mu = 0.$$

where we have assumed $p, q$ possess the stated derivatives. Find the adjoint equation for

**(a.)** *constant coefficient case:* $ay'' + by' + cy = 0$

**(b.)** *Bessel Eqn. of order* $\nu$: $x^2y'' + xy' + (x^2 - \nu^2)y = 0$

**(c.)** *The Airy Equation:* $y'' - xy = 0$

**PP 118** Consider the differential equation $y''' - 3y'' + 2y' = g(t)$. Is $\{1, e^t, e^{2t}\}$ a fundamental solution set ? Explain your answer.

**PP 119** Let $y_1(x) = x^3$ and $y_2(x) = |x|^3$. Show that $W(y_1, y_2)(x) = 0$ for all $x \in \mathbb{R}$. However, explain why $\{y_1, y_2\}$ is linearly independent on $\mathbb{R}$. Does there exist a linear ODE for which $\{y_1, y_2\}$ forms the fundamental solution set? Discuss.

**PP 120** Solve

**(a.)** $y'' + 5y' + 6y = 0$,

**(b.)** $y'' + 4y' + 4y = 0$,

**(c.)** $y'' + 4y' + 5y = 0$.

**PP 121** Solve

**(a.)** $y'' - 36y = 0$ subject the initial conditions $y(0) = 1, y'(0) = 0$,

**(b.)** $y'' + 25 = 0$ subject the initial conditions $y(0) = 1, y'(0) = 0$.

**PP 122** Solve, here $D = d/dx$

**(a.)** $D^2(D^2 - 9)[y] = 0$,

**(b.)** $(D^2 + 6D + 18)^2[y] = 0$,

**(c.)** $(D^2 + 3D + 2)(D^2 - 4)[y] = 0$

**PP 123** Give constant coefficient ODEs for which the following form general solutions. Please leave your answer in $D = d/dx$ factored notation. No need to multiply them out.

**(a.)** $y = c_1e^{-4x} + c_2e^{-3x}$,

**(b.)** $y = c_1e^{10x} + c_2xe^{10x}$

**(c.)** $y = A\cosh(3x + B)$

**(d.)** $y = c_1 + Ae^{2x}\sin(3x + \phi)$

**PP 124** (fitting initial conditions) Given $x(t) = c_1\cos\omega t + c_2\sin\omega t$ is the general solution to

$$x'' + \omega^2x = 0.$$

Show $x(0) = x_o$ and $x'(0) = x_1$ implies $c_1 = x_o$ and $c_2 = x_1/\omega$.

**PP 125** (reduction of order) Use the reduction of order formula $y_2 = y_1 \int \frac{exp\left(-\int p dx\right)}{y_1^2} dx$ to calculate a second linearly independent solution for $x^2 + 2xy' - 6y = 0$ given $y_1 = x^2$.

**PP 126** (reduction of order) Consider $x^2y'' - 3xy' + 5y = 0$ for $x > 0$. You are given that $y_1 = x^2 \cos \ln x$ is a solution. Find $y_2$ for which $y_1, y_2$ forms a fundamental solution set for the given differential equation. One approach is to use the $n = 2$ reduction of order formula as derived in 3.6 of my notes.

**PP 127** (based on Cook section 3.7) Suppose $T = D$ and $S = 3 - x^2 D$. Solve

**(a.)** $ST[y] = 0$,

**(b.)** $TS[y] = 0$.

**PP 128** Consider $f(x) = x^{2+3i}$ for $x > 0$. Find $u, v$ such that $f = u + iv$. Furthermore, by differentiation of $u, v$ show that $f'(x) = (2 + 3i)x^{1-3i}$.
*(the point: you can replace* 2 *with* $a \in \mathbb{R}$ *and* 3 *with* $b \in \mathbb{R}$ *and derive*

$$\frac{d}{dx} x^{a+ib} = (a + ib)x^{a-1+ib};$$

*we see the power rule extends naturally to the case of a complex exponent of the power function. This is an important fact as we deal with solving the Cauchy Euler problem* $ax^2y'' + bxy' + cy = 0$*)*

**PP 129** Solve the following Cauchy Euler problems

**(a.)** $4x^2y'' + y = 0$,

**(b.)** $25x^2y'' + 25xy' + y = 0$

**(c.)** $x^3y''' - 6y = 0$

**PP 130** Suppose that $y_1$ is a nontrivial solution of $y'' + p(x)y' + q(x)y = 0$. We seek a method to derive a second LI solution. Let $y_2$ be such a solution and **show that it must satisfy**

$$\frac{d}{dx}\left[\frac{y_2}{y_1}\right] = \frac{W(y_1, y_2)}{y_1^2}.$$

Now, use Abel's formula to find a nice formula for $y_2$.

**PP 131** (from page 103 of Boyce and DiPrima's 3rd Ed.) Consider for $N \in \mathbb{N}$,

$$xy'' - (x + N)y' + Ny = 0.$$

**(a.)** show $y_1 = e^x$ is a solution.

**(b.)** show that $y_2 = ce^x \int x^N e^{-x} \, dx$ is a second solution. (perhaps use the result of the previous problem, or the theorem from my notes or Ritger & Rose)

**(c.)** set $c = \frac{-1}{N!}$ and show by induction that $y_2(x) = T_n(x)$ where $T_n(x)$ denotes the $n$-th order Taylor polynomial of $e^x$ centered at zero.

**PP 132** Find minimal annihilators for each of the functions below:

**(a.)** $f_1(x) = x^2 e^x$

**(b.)** $f_2(x) = e^x \cos(4x)$

**(c.)** $f_3(x) = x^3 + e^x \cos(4x)$

**(d.)** $f_4(x) = \cos^2(3x) + e^x \cosh(x)$

Now, given what you've just thought through, set-up, but do not determine explicitly, the particular solutions for:

**(a.)** $y'' - 2y' + y = x^2 e^x$

**(b.)** $y'' + 16y = e^x \cos(4x)$

**(c.)** $y''' + y' = x^3 + e^x \cos(4x)$

**(d.)** $y''' + 36y' = \cos^2(3x) + e^x \cosh(x)$

**PP 133** (Zill section 4.4 problem 17) Solve $y'' - 2y' + 5y = e^x \cos(2x)$.

**PP 134** Solve $y'' + 3y' + 2y = t^2$ subject the initial conditions $y(0) = 1$ and $y'(0) = 0$.

**PP 135** (Zill section 4.5 problem 63) Solve $y^{(4)} - 2y''' + y'' = e^x + 1$.

**PP 136** (Zill section 4.6 problem 8) Solve $y'' - y = \sinh(2x)$.

**PP 137** (Zill section 4.6 problem 14) Solve $y'' - 2y' + y = e^t \tan^{-1}(t)$.

**PP 138** Solve $y'' + 3y' + 2y = x + e^{-x} + e^{3x}$.

**PP 139** Solve $y'' + 2y' + y = \frac{e^{-x}}{x+1}$.

**PP 140** Find integral solutions for $y''' + 16y' = f$. (you need to use variation of parameters, I would explicitly calculate the determinants of $S_1, S_2, S_3$ as I discuss in the notes)

**PP 141** Solve $x^2y'' - (x^2 + 2x)y' + (x+2)y = x^3$. Note $y_1 = x$ is a fundamental solution of the DEqn. *Hint: find the 2nd. fundamental soln. and then use variation of parameters to find* $y_p$*...*

**PP 142** Solve $(xD + 3)(D^2 - 4)[y] = 0$. Be careful.

**PP 143** Suppose $L$ is a linear differential operator. Furthermore, suppose $L[y_1] = g_1$ and $L[y_2] = 2g_1$. Solve $L[y] = 0$ using the given solutions.

**PP 144** Find an integral solution for $y''+y = g$ with $y(0) = y_o$ and $y'(0) = y_1$ and $g$ is some integrable function of time $t$.

**PP 145** Consider a third order linear differential equation for which $\sin(x), \cos(x)$ and $\ln(x)$ appear as the fundamental solution set. Call this differential equation $L[y] = 0$. Solve $L[y] = 42$ via variation of parameters. It is interesting to note that even though I asked you to supply an explicit linear ODE $L[y] = 0$ to solve you should not need that explicit formula to solve $L[y] = 42$.

**PP 146** Green's function for a linear ODE $L[y] = f$ provides a method for solving the DEqn via integration. If we assume the initial conditions of the given ODE are all trivial then the operator $L$ can be inverted; $L[y] = f$ with trivial initial conditions iff $y = L^{-1}[f]$. In particular, if $G(x,t)$ is a function for which $y(x) = \int_{x_o}^{x} G(x,t)f(t)dt$ is a solution of $L[y] = f$ then we say $G$ is a **Green's function** for $L$.

In the case of a second order differential equation with fundamental solutions $y_1, y_2$ ( with $L[y_1] = 0$ and $L[y_2] = 0$ for LI $y_1, y_2$ ) we can construct a Green's function as follows:

$$G(x,t) = \frac{y_1(t)y_2(x) - y_1(x)y_2(t)}{y_1(t)y_2'(t) - y_2(t)y_1'(t)}$$

Then observe $y = \int_{x_o}^{x} G(x,t)f(t)dt$ gives a solution to $L[y] = f$ by variation of parameters. Find Green's function for the following solution sets and write an integral solution for $L[y] = f$ for the given $L$ and given initial conditions:

**(a.)** $L = D^2 + 9$, $y_1 = \cos 3t$ and $y_2 = \sin 3t$ with $y(0) = y'(0) = 0$,

**(b.)** $L = D^2 + 3D + 2$, $y_1 = e^{-x}$, $y_2 = e^{-2x}$ with $y(0) = -1$ and $y'(0) = 0$,

**PP 147** Use the Green's function technique to solve

$$y'' + 3y' + 2y = \sin(e^x)$$

subject $y(0) = -1$ and $y'(0) = 0$. In other words, work out the integrals for part (b.) of the previous problem given that $f(x) = \sin(e^x)$.

**PP 148** Suppose a spring is attached to a mass of 1 $kg$ and the spring has spring constant 16 $N/m$. This spring mass system is immersed in an oil which gives a retarding frictional force of $F_{retard} = -\beta v$ where $v$ is velocity and $\beta = 10\ Ns/m$. Find the equations of motion ( please omit units, so in the usual notation $m = 1$, $k = 16$ and $\beta = 10$ ) in the cases

**(a.)** $x(0) = -1$ and $x'(0) = 0$

**(b.)** $x(0) = -1$ and $x'(0) = 12$

**PP 149** Newton's Law for a retarded spring-mass system with external force $f$ yield

$$m\ddot{x} + \beta\dot{x} + kx = f$$

Given $m = 2$, $b = 0$, $k = 32$ and $f = 68e^{-2t}\cos(4t)$ find the equation of motion given the system has initial conditions $x(0) = \dot{x}(0) = 0$.

**PP 150** Consider Newton's Second Law for mass-spring system under a sinusoidal force:

$$\ddot{x} + \omega^2 x = F_o \cos \gamma t$$

given $x(0) = \dot{x}(0) = 0$. Here $F_o, \omega\gamma$ are nonzero constants.

**(a.)** Find $x(t)$ given that $\gamma \neq \omega$

**(b.)** Calculate $x_r(t) = \lim_{\gamma \to \omega} x(t)$

**(c.)** Contrast the motion of $x(t)$ and $x_r(t)$ as $t \to \infty$

**PP 151** Kirchoff's Voltage Law for an RLC-circuit with voltage source $\mathcal{E}$ is given by

$$L\frac{dI}{dt} + RI + \frac{1}{C}Q = \mathcal{E}$$

Since $I = \frac{dQ}{dt}$ we find $L\ddot{Q} + R\dot{Q} + Q/C = \mathcal{E}$. Given that $L = 1$ and $R = 2$ and $C = 0.25$ and $\mathcal{E} = 50\cos t$ find the charge $Q$ as a function of time $t$ given the initial charge and current are both zero for $t = 0$.

**PP 152** If we study the motion of an spring

$$m\ddot{x} + \beta\dot{x} + kx = F$$

such that $\beta^2 - 4mk < 0$ then it is known as **underdamped motion**. If the external force $F = F_o\cos(\gamma t)$ then we find the motion is dominated by the particular solution as $t \to \infty$. Let $\omega = \frac{\sqrt{4mk-\beta^2}}{2m}$, then the homogeneous solution $x_h(t) = e^{\frac{-\beta t}{2m}}\left(c_1\cos(\omega t) + c_2\sin(\omega t)\right) \to 0$ as $t \to \infty$. Show that the particular solution of such a system is given by

$$x_p = \frac{F_o\sin(\gamma t + \phi)}{\sqrt{(k - m\gamma^2)^2 + \beta^2\gamma^2}}$$

where $\phi$ is a constant. Then, find the frequency $\gamma$ which maximizes the magnitude of $x_p$ in the following cases:

**(a.)** $m = 1/2$ and $k = 19$ and $\beta = 1$

**(b.)** $m = 1$ and $k = 2$ and $\beta = \sqrt{6}$.

**PP 153** Find general solution of $y'' - 3y' + 2y = 0$ where $y' = dy/dx$.

**PP 154** Find general solution of $y'' - 6y' + 9y = 0$ where $y' = dy/dx$.

**PP 155** Find general solution of $y'' + 6y' + 13y = 0$ where $y' = dy/dt$.

**PP 156** Find general solution of $(D-2)^3(D^2-1)D^2[y] = 0$ where $D = d/dx$.

**PP 157** Suppose $D = d/dx$ and $L = D^n + a_{n-1}D^{n-1} + \cdots + a_2D^2 + a_1D + a_o$ defines differential equation $L[y] = 0$. Find smallest $n$ and the coefficients $a_o, a_1, \dots, a_{n-1} \in \mathbb{R}$ for which $e^x\cos(2x)$ and $x^3$ are solutions to the differential equation $L[y] = 0$.

**PP 158** Find general solution of $y'' - 9y = t^2 + e^t + 1$.

**PP 159** Solve $y'' + y = 2\cos t + \sin t$

**PP 160** Solve $y'' + 4y = \tan(2x)$.

**PP 161** Find general solution of $y'' + 3y' + 2y = t + 1$.

**PP 162** Solve $y'' + y = \cos t + e^t$

**PP 163** Solve $y'' - 6y + 9y = 0$ where $y' = dy/dt$.

**PP 164** Solve $((D+3)^2+36)[y]=0$ where $D=d/d\theta$.

**PP 165** Let $D=d/dx$. Observe $(D^4+9D^2)[y]=x+\cos(x)$ can be solved by the method of undetermined coefficients aided by the annihilator method. We find the minimal particular solution derived from the annhilator method is: (circle one answer)

**(a.)** $y_p = Ax+B+C\cos(x)+D\sin(x)$

**(b.)** $y_p = Ax^3+Bx^2+Cx\cos(x)+Dx\sin(x)$

**(c.)** $y_p = Ax^3+Bx^2+C\cos(x)+D\sin(x)$

**(d.)** $y_p = A+Bx+C\cos(3x)+D\sin(3x)$

**PP 166** A spring has mass $m=1$, coefficient of damping $\beta=4$ and a spring constant $k=5$. Find the general solution of Newton's Second Law.

**PP 167** Solve $y'-3y=2x+3$.

**PP 168** Find a particular solution for $y''-4y'+3y=65\cos(2t)$.

**PP 169** Find a particular solution of $y''-4y'+3y=e^t$.

**PP 170** Find the general solution of $y''-4y'+3y=130\cos(2t)+7e^t$.

**PP 171** Suppose $L[y]=0$ is an $n$-th order differential equation where $L=D^n+a_{n-1}D^{n-1}+\cdots+a_1D+a_o$ and $D=d/dt$ and $a_{n-1},\dots,a_1,a_o\in\mathbb{R}$. If $L[\,e^t\cos(2t)\,]=0$ and $L[\,t^3e^{-t}\,]=0$ then find the smallest $n$ which allows these solutions and give the explicit form of $L$ in terms of $D=d/dt$. You need not multiply out the formula, I am perfectly happy with $L$ in factored form.

**PP 172** Find an integral solution for $x>0$ to the Cauchy Euler problem
$x^2y''+xy'+9y=g$ where $g$ is a continuous function.

**PP 173** Solve the following differential equations:

**(a.)** $y''-8y'+7y=0$ where $y'=dy/dt$,

**(b.)** $z''+10z+25z=0$ where $z'=dz/dx$,

**(c.)** $u''+7u=0$ where $u'=du/dt$,

**(d.)** $y''+10y'+41y=0$ where $y'=dyd/dx$

**(e.)** $y'''+4y''+5y'=0$ where $y'=dy/dx$.

**PP 174** Find the minimal annihilator for each of the following functions: for each define $D$ as either $D=d/dx$ or $D=d/dt$ as appropriate:

**(a.)** $g=e^x+\sin(4x)$

**(b.)** $g=x^2+\cosh(x)$

**(c.)** $g=te^{-3t}+2$

**(d.)** $g=\cos(x)\sin(3x)$

**(e.)** $g = e^t \cos(6t)$

**PP 175** Set-up, but do not explicitly determine the coefficients, the form of $y_p$ via the method of annihilators. Notice you found the annihilators in the previous problem.

**(a.)** $y'' - y = e^x + \sin(4x)$

**(b.)** $y'' + y' = x^2 + \cosh(x)$

**(c.)** $y'' + 3y' = te^{-3t} + 2$

**(d.)** $y'' + 4y = \cos(x)\sin(3x)$

**(e.)** $y'' + 36y = e^t \cos(6t)$

**PP 176** Solve $y'' - 4y' = 6t + e^t$.

**PP 177** Solve $y'' + 2y' + y = \cos(x) + 3$ subject the initial conditions $y(0) = 0$ and $y'(0) = 1$.

**PP 178** Consider $mx'' + bx' + kx = 0$ where $m > 0$ and $b, k \geq 0$. Show that in every possible case the motion of the solution is bounded.

**PP 179** Find the general solution of

**(a.)** $y'' + \ y = 3\cos(2x)$

**(b.)** $y'' + \ y = \csc(x)$

**(c.)** $y'' + \ y = 2\csc(x) + \cos(2x)$

**PP 180** These require variation of parameters technique.

**(a.)** Solve $y'' - 2y' + y = \frac{1}{t}e^t$

**(b.)** Solve $y'' + y = \sec^3\theta$.

**PP 181** Solve the integral $\int (x^3 + 2x)e^x \, dx = y$ by solving $\frac{dy}{dx} = (x^2 + 2x)e^x$ via the method of undetermined coefficients

**PP 182** Consider the differential equation given by: $D = d/dx$ and

$$(D^4 + 2D^3 + 10D^2 + 18D + 9)[y] = 0$$

You are given that $y = \sin 3x$ is a solution to the above. Use this data to help solve the problem.