Name: (please print name here $\rightarrow$) .

MATH 334:	MISSION 2: n-TH ORDER DEQNS [50PTS]

You write the solution neatly in the box (or space) provided and show work on your own, standard sized, non-fuzzy edged, paper. The work must be labeled with problem number and part as appropriate and if a problem is skipped it must be mentioned in the attached work. Each part 1pt. Formatting is 5pts. There are 50pts that can be earned. Enjoy.

Problem 21 Consider the differential equation $y''' - 3y'' + 2y' = g(t)$. Is $\{1, e^t, e^{2t}\}$ a fundamental solution set ? Explain your answer (whole answer goes below).

Problem 22 Let $y_1(x) = x^3$ and $y_2(x) = |x|^3$. Show that $W(y_1, y_2)(x) = 0$ for all $x \in \mathbb{R}$. However, explain why $\{y_1, y_2\}$ is linearly independent on $\mathbb{R}$. Does there exist a linear ODE for which $\{y_1, y_2\}$ forms the fundamental solution set? Discuss. (whole answer goes below)

Problem 23 Consider $f(x) = x^{a+ib}$ for $x > 0$ and $a, b \in \mathbb{R}$. Find u, v such that $f = u + iv$. Furthermore, by differentiation of u, v, derive $\frac{d}{dx}x^{a+ib} = (a+ib)x^{a-1+ib}$. (whole answer goes below)

Problem 24 (IVP) Solve $y'' + 2y' + y = 0$ where $y(0) = 1$ and $y'(0) = -3$ given $y' = dy/dx$.

Problem 25 (IVP) Solve $(D^2 - 2D + 2)[y] = 0$ given $y(\pi) = e^{\pi}$ and $y'(\pi) = 0$ and $D = d/dt$.

Problem 26 (Constant Coefficient Problems) Find the general solution for each DEqn below: assume the independent variable in each solution is denoted by x.

(a.) $y'' - y' - 11y = 0$

(b.) $4w'' + 20w' + 25w = 0$

(c.) $y'' - 8y' + 7y = 0$

(d.) $z'' + 10z' + 25z = 0$

(e.) $u'' + 7u = 0$

(f.) $y'' + 10y' + 41y = 0$

(g.) $y''' + 2y'' - 8y' = 0$

(h.) $u''' - 9u'' + 27u' - 27u = 0$

(i.) $y^{(4)} + 4y'' + 4y = 0$

(j.) $(D^4 - 36)[y] = 0$

(k.) $((D+1)^2 + 36)^2[y] = 0$

Problem 27 Solve $y^{(4)} + 2y''' + 10y'' + 18y' + 9y = 0$ given that $y = \sin(3x)$ is a solution.

Problem 28 Use the method of annihilators to set-up (but do not determine explicitly) the particular solutions for:

(a.) $y'' - 2y' + y = x^2e^x$

(b.) $y'' + 16y = e^x\cos(4x)$

(c.) $y''' + y' = x^3 + e^x\cos(4x)$

(d.) $y''' + 36y' = \cos^2(3x) + e^x\cosh(x)$

Problem 29 (nonhomogeneous problems) Find the general solution for each DEqn below:

(a.) $y'' - y = 1 - 11t$.

(b.) $y'' - 9y = t^2 + e^t + 1$.

(c.) $y'' + 3y' + 2y = t + 1$.

(d.) $y'' + y = \cos t + e^t$

Problem 30 Solve $z'' + z = 2e^{-x}$ given $z(0) = 0$ and $z'(0) = 0$.

Problem 31 Solve $y'' + 2y' + y = \dfrac{e^{-x}}{x+1}$.

Problem 32 Solve $y'' + 4y = \tan(2x)$.

Problem 33 Solve $y'' + 3y' + 2y = \sin(e^x)$

Problem 34 Solve the following cauchy euler problems. Give your solution as a real linear combination of the real-valued functions in the fundamental solution set.

(a.) $4x^2y'' + y = 0$

(b.) $x^2y'' - 3xy' + 5y = 0$

(c.) $2x^2y'' + 3xy' - y = 0$

(d.) $x^3y''' + 2x^2y'' - xy' + y = 0$

Problem 35 Find an integral solution for $x > 0$ to the Cauchy Euler problem $x^2y'' + xy' + 9y = g$ where g is a continuous function.

Problem 36 Suppose $T = D$ and $S = 3 - x^2D$. Solve

(a.) $ST[y] = 0$,

(b.) $TS[y] = 0$.

Problem 37 A spring has mass $m = 1$, coefficient of damping $\beta = 4$ and a spring constant $k = 5$. Find the general solution of Newton's Second Law.

Problem 38 Newton's Law for a retarded spring-mass system with external force f yield

$$m\ddot{x} + \beta\dot{x} + kx = f$$

Given $m = 2$, $\beta = 0$, $k = 32$ and $f = 68e^{-2t}\cos(4t)$ find the equation of motion given the system has initial conditions $x(0) = \dot{x}(0) = 0$.

Problem 39 Consider Newton's Second Law for mass-spring system under a sinusoidal force:

$$\ddot{x} + \omega^2 x = F_o \cos\gamma t$$

given $x(0) = \dot{x}(0) = 0$. Here $F_o, \omega\gamma$ are nonzero constants.

(a.) Find $x(t)$ given that $\gamma \neq \omega$

(b.) Calculate $x_r(t) = \lim_{\gamma\to\omega} x(t)$

Problem 40 Kirchoff's Voltage Law for an RLC-circuit with voltage source $\mathcal{E}$ is given by

$$L\frac{dI}{dt} + RI + \frac{1}{C}Q = \mathcal{E}$$

Since $I = \frac{dQ}{dt}$ we find $L\ddot{Q} + R\dot{Q} + Q/C = \mathcal{E}$. Given that $L = 1$ and $R = 2$ and $C = 0.25$ and $\mathcal{E} = 50\cos t$ find the charge Q as a function of time t given the initial charge and current are both zero for $t = 0$.

Problem 41 If we study the motion of an spring

$$m\ddot{x} + \beta\dot{x} + kx = F$$

such that $\beta^2 - 4mk < 0$ then it is known as **underdamped motion**. If the external force $F = F_o \cos(\gamma t)$ then we find the motion is dominated by the particular solution as $t \to \infty$. Let $\omega = \frac{\sqrt{4mk-\beta^2}}{2m}$, then the homogeneous solution $x_h(t) = e^{\frac{-\beta t}{2m}}(c_1\cos(\omega t) + c_2\sin(\omega t)) \to 0$ as $t \to \infty$. It can be shown that the particular solution of such a system is given by

$$x_p = \frac{F_o \sin(\gamma t + \phi)}{\sqrt{(k - m\gamma^2)^2 + \beta^2\gamma^2}}$$

where ϕ is a constant. Find the frequency γ which maximizes the magnitude of x_p in the following cases:

(a.) $m = 1/2$ and $k = 19$ and $\beta = 1$

(b.) $m = 1$ and $k = 2$ and $\beta = \sqrt{6}$.

Problem 42 Consider the differential equation $x^2y'' - 3xy' + 5y = 0$. You are given that $y_1 = x^2\cos(\ln(x))$ serves as a solution. Derive the second LI solution to the differential equation via a theorem we developed in lecture.

Problem 43 Solve $x^2y'' - (x^2 + 2x)y' + (x + 2)y = x^3$ given that $y_1 = x$ is a fundamental solution of the differential equation.