Name: (please print name here $\rightarrow$) .

Math 334: Mission 4: Integral Curves, Energy, Laplace Transform Technique

While complex solutions may be useful as in-between work the solutions requested on this assignment are real solutions. Please refer to Mission 1 for formatting rules. Thanks!

Problem 73 Determine if the vector field $\vec{F}$ is conservative on S. If so, find the potential energy function U for which $\vec{F} = -\nabla U$.

(a.) $\vec{F} = \langle x - 7x^6y^7, y^2 - 7x^7y^6 \rangle$ for $S = \mathbb{R}^2$

(b.) $\vec{F} = \dfrac{1}{x^2+y^2}\langle x, y \rangle$ for $S = \mathbb{R} - \{(0,0)\}$

(c.) $\vec{F} = \dfrac{1}{x^2+y^2}\langle -y, x \rangle$ for $S = \{(x,y) \mid x > 0\}$

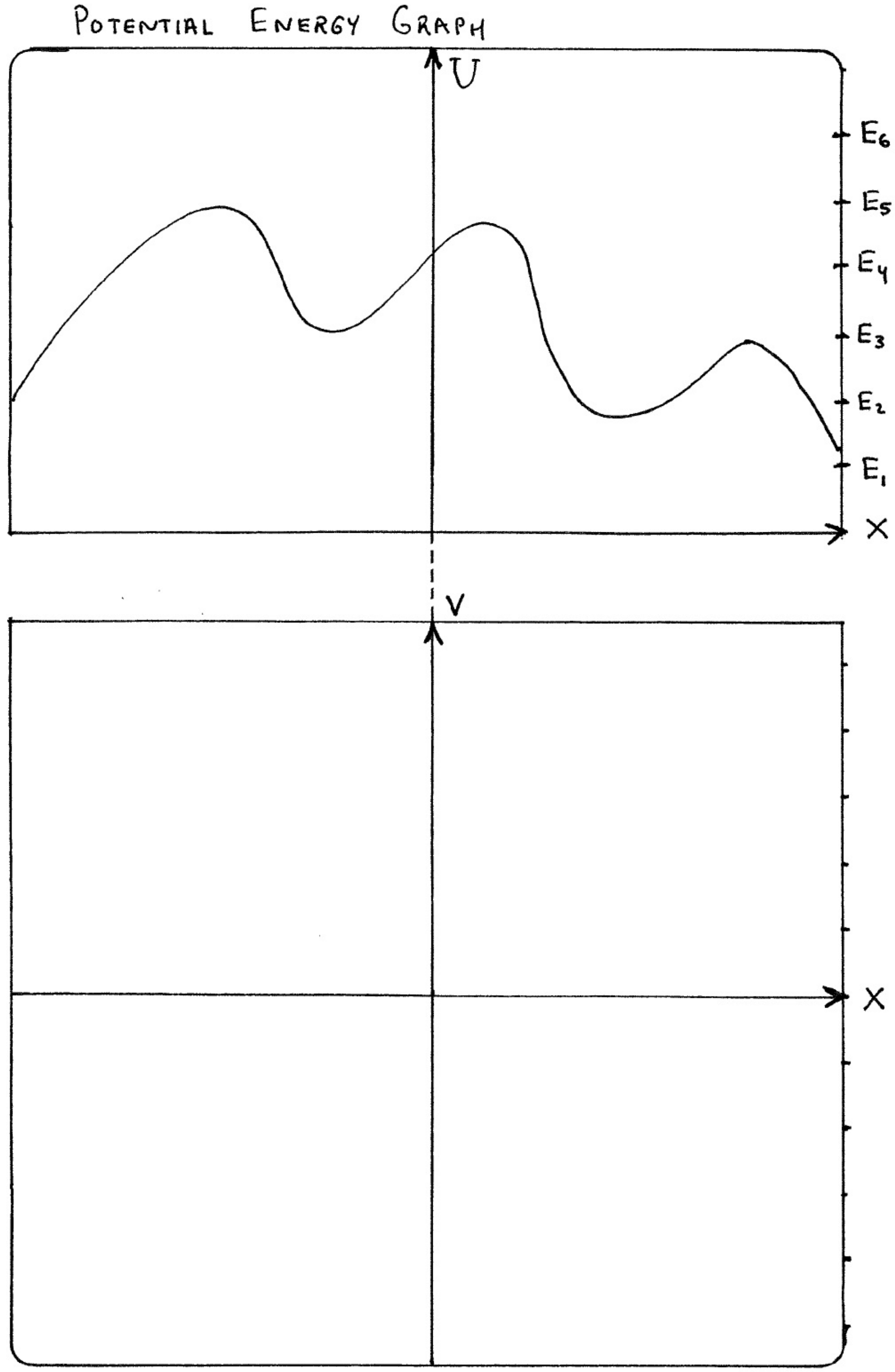
(d.) $\vec{F} = \dfrac{1}{x^2+y^2}\langle -y, x \rangle$ for $S = \mathbb{R}^2 - \{(0,0)\}$

Problem 74 Solve the following exact equations:

(a.) $(x - 7x^6y^7)dx + (y^2 - 7x^7y^6)dy = 0,$

(b.) $\dfrac{xdx + ydy}{x^2+y^2} = 0.$

Problem 75 Plot the phase plane (or Poincare plot) given the potential energy plot below. For each energy $E_1, E_2, \ldots, E_6$ graph the corresponding trajectories below. Use a couple different colors so your work is easy to follow. Be neat. If no motion is possible then explain why.



Problem 76 For each vector field below find the parametrization of the integral curve $\vec{\gamma} : \mathbb{R} \to \mathbb{R}^2$ for which $\frac{d\vec{\gamma}}{dt} = \vec{F}(\vec{\gamma}(t))$. In particular, for $\vec{F} = \langle P, Q \rangle$ if we write $\vec{\gamma} = \langle x, y \rangle$ then we wish to solve $\frac{dx}{dt} = P(x, y)$ and $\frac{dy}{dt} = Q(x, y)$.

(a.) $\vec{F} = \langle 3x + y, 3y \rangle$

(b.) $\vec{F} = \langle 2x + y, x + 2y \rangle$

(c.) $\vec{F} = \langle -2y, 2x \rangle$

Problem 77 For the vector fields of the previous problem, solve the differential equation $\frac{dy}{dx} = \frac{Q}{P}$ and show that the Cartesian solution found here is parametrized by your solution of the previous problem.

(a.) $P = 3x + y$, $Q = 3y$

(b.) $P = 2x + y$, $Q = x + 2y$

(c.) $P = -2y$, $Q = 2x$

Problem 78 Calculate $\mathcal{L}\{2t^2e^{-t}\}(s)$

Problem 79 Calculate $\mathcal{L}\{\sin(2t)\cos(3t)\}(s)$

Problem 80 Calculate $\mathcal{L}\{te^t\sin(2t+3)\}(s)$

Problem 81 Calculate the inverse Laplace transforms below:

(a.) $\mathcal{L}^{-1}\left\{\frac{6}{(s-1)^4}\right\}(t)$

(b.) $\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}(t)$

(c.) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\}(t)$

(d.) $\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\}(t)$

Problem 82 Let $F(s) = \dfrac{3s - 15}{2s^2 - 4s + 10}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 83 Let $F(s) = \dfrac{6s^2 - 13s + 2}{s(s-1)(s-6)}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 84 Let $F(s) = \dfrac{7s^3 - 2s^2 - 3s + 6}{s^3(s-2)}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 85 Let $F(s) = \tan^{-1}\left(\dfrac{1}{s}\right)$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 86 Solve $y'' + 6y' + 5y = 12e^t$ with $y(0) = -1$ and $y'(0) = 7$ via the Laplace transform technique.

Problem 87 Let $g(t) = \begin{cases} 0, & t < 1 \\ t - 1, & 1 < t < 2 \\ 3 - t, & 2 < t < 3 \\ 0, & 3 < t \end{cases}$. Calculate $G(s)$.

Problem 88 Calculate the Laplace transforms of the following functions

(a.) $f(t) = \sin(t)\cos(2t) + \sin^2(3t)$

(b.) $f(t) = e^t u(t-3) + \sin(t)u(t-6)$

Problem 89 Solve $y'' + 4y' + 4y = u(t-\pi) - u(t-2\pi)$ with $y(0) = 0$ and $y'(0) = 0$ via the Laplace transform technique.

Problem 90 Solve $y'' + 5y' + 6y = g(t)$ given $y(0) = 0$ and $y'(0) = 2$ where $g(t) = \begin{cases} 0, & 0 < t < 1 \\ t, & 1 < t < 5 \\ 1, & 5 < t \end{cases}$.

Problem 91 Let $F(s) = \dfrac{e^{-\pi s}s}{s^2 + 6s + 13}$. Calculate the inverse Laplace transform of $F(s)$.

Problem 92 Solve $w'' + w = \delta(t-\pi)$ where $w(0) = 0$ and $w'(0) = 0$.

Problem 93 Solve $y'' + y = 4\delta(t-2) + t^2$ given $y(0) = 0$ and $y'(0) = 2$.

Problem 94 A hammer hits a spring mass system at time $t = \pi/2$ and thus Newton's Second Law gives

$$\frac{d^2x}{dt^2} + 9x = -3\delta(t - \pi/2)$$

with $x(0) = 1$ and $x'(0) = 0$ since the spring is initially stretched to 1-unit and released from rest. Calculate the equation of motion and explain what happens after the hammer hits the spring at time $t = \pi/2$.

Problem 95 (Ritger & Rose section 9-6 problem 1a) Use convolution to find the inverse Laplace transform of $\dfrac{1}{s^2(s-a)}$ for $a \neq 0$.

Problem 96 Find an integral solution of $y'' + y = g$ via Laplace transforms and convolution. You may assume g is an integrable function of time t.