Name: (please print name here →) .

MATH 334: MISSION 4: COUPLED SPRINGS, ENERGY, LAPLACE TRANSFORM TECHNIQUE

While complex solutions may be useful as in-between work the solutions requested on this assignment are real solutions. Please refer to Mission 3 for formatting rules. There are 50pts to earn here, including 5pts for completely correct formatting.Thanks!

Problem 68 Suppose two identical masses m are pulled back and forth by three springs with spring constants k_1, k_2, k_3 where the leftmost spring has constant k_1, the middle spring has k_2 and the rightmost spring has constant k_3. If $x_1(t)$ and $x_2(t)$ are the displacements from the respective equilbria of the masses then

(a.) Find the equations of motion for arbitrary m, k_1, k_2, k_3:

(b.) (2pts) Suppose $m = 1$ and $k_1 = 30, k_2 = 10, k_3 = 35$. Solve the equations of motion.

Problem 69 (2pts) Suppose a mass m has net-force $F = -dU/dx$ where U is the potential energy function for the particle which moves one-dimensionally in the x-direction. Prove that total energy is conserved. (show work below)

Problem 70 Determine if the vector field $\vec{F}$ is conservative on S. If so, find the potential energy function U for which $\vec{F} = -\nabla U$. If no such U exists then explain why.

(a.) $\vec{F} = \langle x - 7x^6y^7, y^2 - 7x^7y^6 \rangle$ for $S = \mathbb{R}^2$

(b.) $\vec{F} = \dfrac{1}{x^2+y^2}\langle x, y\rangle$ for $S = \mathbb{R} - \{(0,0)\}$

(c.) $\vec{F} = \dfrac{1}{x^2+y^2}\langle -y, x\rangle$ for $S = \{(x,y) \mid x > 0\}$

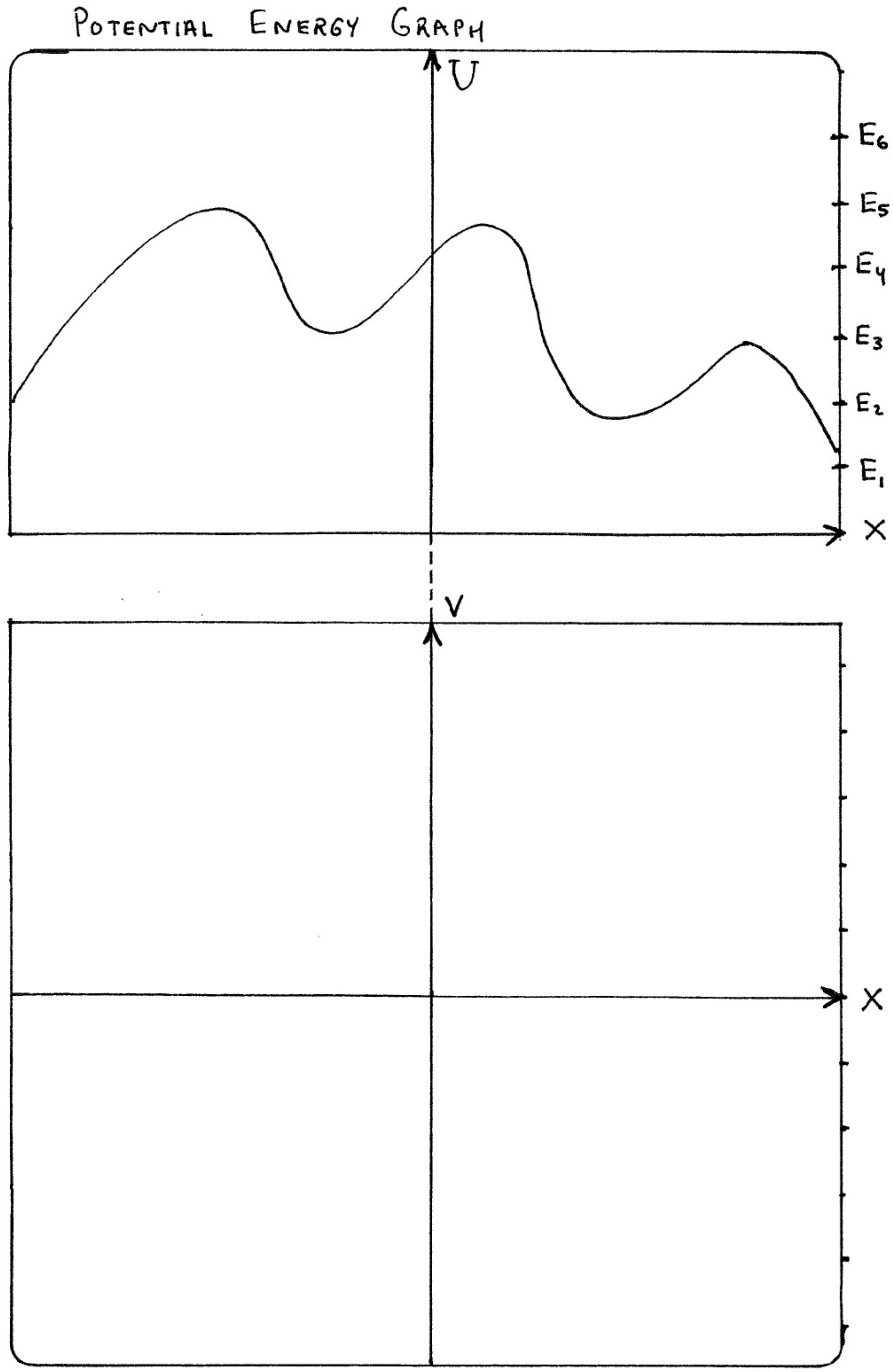
(d.) $\vec{F} = \dfrac{1}{x^2+y^2}\langle -y, x\rangle$ for $S = \mathbb{R}^2 - \{(0,0)\}$

Problem 71 Solve the following exact equations:

(a.) $(x - 7x^6y^7)dx + (y^2 - 7x^7y^6)dy = 0,$

(b.) $\dfrac{xdx + ydy}{x^2+y^2} = 0.$

Problem 72 (2pts) Plot the phase plane (or Poincare plot) given the potential energy plot below. For each energy $E_1, E_2, \ldots, E_6$ graph the corresponding trajectories below. Use a couple different colors so your work is easy to follow. Be neat. If no motion is possible then explain why.



Problem 73 Use the Laplace transforms to solve the integral equation $f(t) + \int_0^t f(\tau)\, d\tau = 1$.

Problem 74 Calculate $\mathcal{L}\{2t^2 e^{-t}\}(s)$

Problem 75 Calculate $\mathcal{L}\{\sin(2t)\cos(3t)\}(s)$

Problem 76 Calculate $\mathcal{L}\{te^t \sin(2t+3)\}(s)$

Problem 77 Calculate the inverse Laplace transforms below:

(a.) $\mathcal{L}^{-1}\left\{\frac{6}{(s-1)^4}\right\}(t)$

(b.) $\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}(t)$

(c.) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\}(t)$

(d.) $\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\}(t)$

Problem 78 Let $F(s) = \dfrac{3s-15}{2s^2-4s+10}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 79 (2pts) Let $F(s) = \dfrac{6s^2-13s+2}{s(s-1)(s-6)}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 80 (2pts) Let $F(s) = \dfrac{7s^3-2s^2-3s+6}{s^3(s-2)}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 81 Let $F(s) = \tan^{-1}\left(\dfrac{1}{s}\right)$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 82 (2pts) Solve $y'' + 6y' + 5y = 12e^t$ with $y(0) = -1$ and $y'(0) = 7$ via the Laplace transform technique.

Problem 83 Calculate the Laplace transforms of the following functions

(a.) $f(t) = \sin(t)\cos(2t) + \sin^2(3t)$

(b.) $f(t) = e^t u(t-3) + \sin(t)u(t-6)$

Problem 84 (2pts) Solve $y'' + 4y' + 4y = u(t-\pi) - u(t-2\pi)$ with $y(0) = 0$ and $y'(0) = 0$ via the Laplace transform technique.

Problem 85 (2pts) Solve $y'' + 5y' + 6y = g(t)$ where $g(t) = \begin{cases} 0, & 0 < t < 1 \\ t, & 1 < t < 5 \\ 1, & 5 < t \end{cases}$ given $y(0) = 0$ and $y'(0) = 2$.

Problem 86 (2pts) Let $F(s) = \dfrac{e^{-\pi s}s}{s^2 + 6s + 13}$. Calculate the inverse Laplace transform of $F(s)$.

Problem 87 (2pts) Solve $w'' + w = \delta(t-\pi)$ where $w(0) = 0$ and $w'(0) = 0$.

Problem 88 (2pts) Solve $y'' + y = 4\delta(t-2) + t^2$ given $y(0) = 0$ and $y'(0) = 2$.

Problem 89 (2pts) A hammer hits a spring mass system at time $t = \pi/2$ and thus Newton's Second Law gives

$$\frac{d^2x}{dt^2} + 9x = -3\delta(t - \pi/2)$$

with $x(0) = 1$ and $x'(0) = 0$ since the spring is initially stretched to 1-unit and released from rest. Calculate the equation of motion and explain what happens after the hammer hits the spring at time $t = \pi/2$.

Problem 90 Use convolution to find the inverse Laplace transform of $\dfrac{1}{s^2(s-a)}$ for $a \neq 0$.

Problem 91 Find an integral solution of $y'' + y = g$ via Laplace transforms and convolution. You may assume g is an integrable function of time t.