Name: (please print name here $\rightarrow$) .

MATH 334:	MISSION 5: SERIES SOLUTION TECHNIQUES & PDES [50PTS]

While complex solutions may be useful as in-between work the solutions requested on this assignment are real solutions. Please refer to Mission 3 for formatting rules. 50 points to earn here, 5 points for completely correct formatting. Thanks!

Problem 92 (1pt) Find the first 4 nonzero terms in the power series solution about $x = 0$ for $z'' - x^2 z = 0$.

Problem 93 (2pt) Find the complete power series solution (including a formula for the general coefficient) about $x = 0$ for:

$$y' - 2xy = 0.$$

Problem 94 (2pt) Find the complete power series solution (including a formula for the general coefficient) about $x = 0$ for:

$$y'' - xy' + 4y = 0.$$

Problem 95 (1pt) Find the first four nonzero terms in the power series solution about $x = 0$ for:

$$y'' - e^{2x}y' + (\cos x)y = 0, \qquad y(0) = -1,\ y'(0) = 1.$$

Problem 96 (1pt) Find the first four nonzero terms in the power series solution about $x = 0$ for:

$$z'' + xz' + z = x^2 + 2x + 1.$$

Problem 97 (1pt) Find the singularities of $x(x^2+2x+2)y'' + (x^2+1)y' + 3y = 0$ and determine the largest open interval of convergence for a solution of the form $y = \sum_{n=0} a_n(x+2)^n$. *Think. Do not try to solve this, I'm asking you about the interval of convergence, I'm* **not** *asking for what a_n are in particular*

Problem 98 (1pt) Suppose we define $e^z = \displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} z^n$. Show that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$.

Problem 99 (1pt) Suppose $\displaystyle\sum_{k=0}^{\infty} (a_{2k}x^{2k} + b_{2k+1}x^{2k+1} = e^x + \cos(x+2)$. Find explicit formulas for a_{2k} and b_{2k+1} via Σ-notation algebra.

Problem 100 (2pt) Find a power series solution to the integrals below:

(a.) $\displaystyle\int \frac{x^3 + x^6}{1 - x^3}\, dx$

(b.) $\displaystyle\int x^8 e^{x^3+2}\, dx$

Problem 101 (3pt) Solve via method of Frobenius: $2x^2y'' + 3xy' - (1 + x)y = 0$.

Problem 102 (3pt) Solve via method of Frobenius: $3x^2y'' + (5x + 3x^3)y' + (3x^2 - 1)y = 0$.

Problem 103 (3pt) Solve via method of Frobenius: $(2x^2 - x^3)y'' + (7x - 6x^2)y' + (3 - 6x)y = 0$.

Problem 104 (3pts) Find all nontrivial solutions to the following Boundary Value Problems (BVPs) on the given interval I. For each case, determine what values of K permit a family of nontrivial solutions. Use $n \in \mathbb{N}$ to index the solutions as appropriate.

(a.) $y'' + Ky = 0$ with $y(0) = y(\pi) = 0$, $I = (0, \pi)$

(b.) $y'' + Ky = 0$ with $y'(0) = y'(1) = 0$, $I = (0, 1)$

(c.) $y'' + Ky = 0$ with $y'(0) = y(2) = 0$, $I = (0, 2)$

Problem 105 (3pt) Find the Fourier series which represents $f(x) = x^2$ on $-\pi < x < \pi$. Use your result to calculate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

Problem 106 (1pt) Find the Fourier sine series which represents $f(x) = x - x^2$ for $0 < x < 1$.

Problem 107 (1pt) Find the Fourier cosine series which represents $f(x) = \pi - x$ for $0 < x < \pi$.

Problem 108 (1pts) Consider separation of variables for the PDE given below

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

given by the proposed product $u(r, \theta) = R(r)T(\theta)$. Show this proposal yields

$$T''(\theta) + \lambda T(\theta) = 0$$

where λ is a constant and find related ODE for R which also involves λ.

Problem 109 (2pts) Solve the heat equation $\dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{\partial x^2}$ on $0 < x < \pi$ for $t > 0$ given the boundary conditions $\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0$ for $t > 0$ and initial condition $u(x,0) = x$ for $0 < x < \pi$.

Problem 110 (2pts) Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ for $0 < x < 1$, $t > 0$ given the boundary conditions $u(0,t) = u(1,t) = 0$ for $t > 0$ and $u(x,0) = x(1-x)$ and $\frac{\partial u}{\partial t}(x,0) = \sin(7\pi x)$ for $0 < x < 1$.

Problem 111 (2pts) Solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ for $0 \leq x, y \leq \pi$ given the boundary conditions $u(0,y) = u(\pi,y) = 0$ for $0 \leq y \leq \pi$ and $u(x,0) = f(x)$ and $u(x,\pi) = 0$ for $0 \leq x \leq \pi$. You can expect the solution to be formed in terms of appropriate integrals involving f based on Fourier analytic argumentation.

Problem 112 (1pts) Let $L(x, y, \dot{x}, \dot{y}) = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + \alpha x^2 - \beta y^2$. Find the Euler-Lagrange equations for x and y.

Problem 113 (1pts) In physics, polar coordinates are sometimes given by $x = s\cos\phi$ and $y = s\sin\phi$. The **kinetic energy** of a mass m undergoing two-dimensional motion is given by $T = \frac{m}{2}(\dot{x}^2+\dot{y}^2)$. Derive the formula for kinetic energy of a mass m in polar coordinates.

Problem 114 (1pts) Find the equations of motion for the polar coordinates of a free particle of mass m. Identify any conserved quantities.

Problem 115 (2pts) In physics the spherical coordinate system is given by $x = r\cos\phi\sin\theta$, $y = r\sin\phi\sin\theta$ and $z = r\cos\theta$. The **kinetic energy** of a mass m in Cartesian coordinates is given by $T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$. Derive the formula for kinetic energy of a mass m in spherical coordinates.

Problem 116 (2pts) Find the equations of motion for the spherical coordinates of a free particle. Identify any conserved quantities.

Problem 117 (2pts) Suppose that the net-force on a given mass m is a conservative force which depends only on the distance from the origin. In this case, $F = -\frac{dU}{dr}$ and we can express the Lagrangian $L = T - U$ using the formula you derived in the previous problem and the arbitrary potential $U = U(r)$. Derive the Euler Lagrange equations for r, θ and ϕ.