

Don't even think about working these problems out on this page alone. The solutions should be written neatly on lined or unlined paper with the work clearly labeled. Do not omit scratch work. I need to see all steps. Thanks and enjoy.

Problem 24 [25pts] Calculate the Laplace transforms of the following functions using the table of basic Laplace transforms plus possibly the given Theorems. Derive any needed trigonometric identities via the imaginary exponential tricks, you may assume the double-angle and adding angle formulas are known. Just keep in mind I expect you to be able to do Problems similar to these without technology so you best find a way to manage the trigonometry either through memorization or derivation.

(a.) $f(t) = \sin(t) \cos(2t)$

(b.) $f(t) = \sin^2(3t)$

(c.) $f(t) = \sin^2(t + 3)$

(f.) $f(t) = \begin{cases} t, & 0 \leq t \leq 2 \\ \sin(t) & t > 2 \end{cases}.$

(g.) $f(t) = te^{-2t} + t \sin(t)$

Problem 25 [5pts] Solve the following differential equation with the given initial condition by the method of Laplace transforms.

$$\frac{dy}{dx} + 2y = f(x), \quad y(0) = 0, \quad \text{with} \quad f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ -1, & x > 1 \end{cases}$$

notice you have already solved this without Laplace transforms back in Problem 5 so you can check your answer.

Problem 26 [25pts] Solve the following differential equations with the given initial conditions by the method of Laplace transforms.

(a.) $y'' + y' - 2y = 0$ where $y(0) = 2$ and $y'(0) = 1$

(b.) $y'' - 2y' + y = \delta(t - 2)$ where $y(0) = 1$ and $y'(0) = 0$

(c.) $y'' - 8y' + 7y = u(t - 2)$ where $y(0) = 0$ and $y'(0) = 0$

(d.) $y'' + y = \delta(t - 2)$ where $y(0) = 1$ and $y'(0) = 2$

(e.) $y'' - 8y' + 7y = u(t - 2) + u(t - 4)$ where $y(0) = 0$ and $y'(0) = 0$

Problem 27 [20pts] Find the series solution centered about zero for the following differential equations. Give the best answer possible for the solution, if you identify the series as a known elementary function then great, if you can find a general formula for the n-th term in the series good, if all you can do is find the first 6 nontrivial terms in the solution then ok. Best answer best credit. (see E159 and E160, but keep in mind it's not always possible to find an elementary function to represent the series.)

(a.) $(x - 1)y'' - xy' + y = 0,$

(b.) $y'' - 2xy' + y = 0,$

(c.) $(x - 1)y'' + y' = 0$

(d.) $(x - 1)y'' - xy' + y = 0$ subject to initial conditions $y(0) = -2, y'(0) = 6$

Problem 28 [5pts] Find all singular points of the differential equation below (real or complex). If the singular point is regular explain why that is the case.

$$x^3(x^2 - 25)(x - 2)^2y'' + 3x(x - 2)y' + 7(x + 5)y = 0$$

Problem 29 [20pts] Find the solution at the regular singular point zero. Use the Method of Frobenius to find at least the first 6 nontrivial terms in the solution (I want at least 3 nonzero terms from each of the two linearly independent solutions). (please remind me to give you problems later this semester if I forget to update this, Thanks.)

(a.) T.B.A.

(b.) T.B.A.