Don't even think about working these problems out on this page alone. The solutions should be written neatly on lined or unlined paper with the work clearly labeled. Do not omit scratch work. I need to see all steps. Skipping details of integration will result in a loss of credit. Thanks and enjoy.

Problem 30 [10pts] Work out problem #26 of §4.2.

Problem 31 [20pts] Consider the following boundary value problems. Use the separation of variables $u(x,t) = X(x)T(t)$ to find a family of solutions $u_n(x,t) = X_n(t)T_n(t)$. Each $u_n(x,t)$ corresponds to a particular choice of the "eigenvalue" for the problem. Both of these are modified heat equations, the $-4u$ and $-u$ terms indicate an extra loss of temperature which is proportional to the temperature u (they are similar to Newton's Law of Cooling terms). I included parts (a.) and (b.) because the expansions work out different even though they come from similar PDEs. The boundary conditions make the difference. (one is sines, the other is cosines)

(a.)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 4u$$

for $t > 0$ and $0 < x < \pi$ subject to $u(0,t) = 0$ and $u_x(\pi,t) = 0$ for $t > 0$.

(b.)

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} - u$$

for $t > 0$ and $0 < x < \pi$ subject to $u_x(0,t) = 0$ and $u_x(\pi,t) = 0$ for $t > 0$.

Problem 32 [10pts] Find formal solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 4u$ for $t > 0$ and $0 < x < 1$ subject to $u(0,t) = 0$ and $u(\pi,t) = 0$ for $t > 0$ and the initial condition $u(x,0) = 1 - 2x$ for $0 < x < \pi$. You can refer to the result of Problem 30a to get the problem started. (you can follow the same logic as I used in PH-135-138 for example)

Problem 33 [10pts] Find formal solution of $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} - u$ for $t > 0$ and $0 < x < \pi$ subject to $u_x(0,t) = 0$ and $u_x(\pi,t) = 0$ for $t > 0$ and the initial condition $u(x,0) = x^2$ for $0 < x < \pi$. You can refer to the result of Problem 30a to get the problem started. (you can follow the same logic as I used in PH-135-138 for example)

Problem 34 [20pts] Find formal solution of $u_{tt} = u_{xx} - 2$ with $t > 0$ and $0 < x < 1$ subject to boundary conditions $u(0,t) = 1$ and $u(1,t) = 0$ and initial conditions $u(x,0) = x^2$, $u_t(x,0) = 0$. This PDE models a string with gravity pulling down (I am not being careful about units)

Problem 35 [20pts] Find formal solution of Laplace's Equation $u_{xx}+u_{yy}=0$ for a rectangle $0<x<\pi$, $0<y<1$ given the boundary conditions $u_x(0,y)=u_x(\pi,y)=0$ on $0\leq y\leq 1$ and $u(x,0)=\cos(x)-\cos(3x)$ and $u(x,1)=\cos(2x)$ for $0\leq x\leq\pi$. This PDE arises when we study the electric potential in electrostatics. The boundary condition $u_x(0,y)=0$ has the physical interpretation that the x-component of the electric field is zero along the left edge of the rectangle. Boundary conditions in electrostatics follow from the distribution of charge on the system. It is a very interesting story, but our goal here is just to learn enough of the math so we can focus on the physics when we get to it in an applied course. By the way, this is Problem 5 of §10.7 so you can check your answer.

Problem 36 [10pts] Work out problem #16 of §10.6.

Test IV is completely take-home. It consists of the problems on this Problem Set. If you get all of these correct then you will be credited 100 on Test IV automatically. Otherwise, you'll have 2 days to fix your solution once I return the graded Problem Set IV.