Partial Fractions Make-up Quiz

Name: _____

① Given $f(x) = \frac{4}{x^2 - 1}$ which of the following is correct?

a.) $f(x) = \frac{Ax^2 + B}{x^2 - 1}$

b.) $f(x) = \frac{4}{x^2} - \frac{4}{1}$

c.) $f(x) = \frac{2}{x-1} - \frac{2}{x+1}$

d.) $f(x) = \frac{1}{x-1} - \frac{1}{x+1}$

② Given $f(x) = \frac{x^2 + 3x - 11}{x^3(x^2 + x + 1)(x^2 - 1)}$ which of the following is correct?

a.) $f(x) = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{x^2-1}$

b.) $f(x) = \frac{Ax^2+Bx+C}{x^3} + \frac{Dx+E}{x^2+x+1} + \frac{F}{x+1} + \frac{G}{x-1}$

c.) $f(x) = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+x+1} + \frac{E}{x+1} + \frac{G}{x-1}$

③ True or False : $\frac{x^2}{x^2-1} = \frac{Ax+B}{x^2-1}$ for appropriately chosen constants A & B.

④ True or False : $\frac{x^2}{x^2+3} = A + \frac{B}{x^2+3}$; A & B constants.

⑤ True or False : $\frac{x+5}{x^2+5x+6} = \frac{A}{x+3} + \frac{B}{x+2}$; A & B constants.

Partial Fractions Make-Up Quiz Solution

(1) $f(x) = \frac{4}{x^2-1} = \frac{4}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$

$4 = A(x-1) + B(x+1)$

$x=1$ $\quad 4 = 2B \Rightarrow B = 2$

$x=-1$ $\quad 4 = -2A \Rightarrow A = -2$

$\therefore \boxed{f(x) = \frac{-2}{x+1} + \frac{2}{x-1}}$

part C. was correct.

(2.) Given $f(x) = \frac{x^2+3x-11}{x^3(x^2+x+1)(x^2-1)}$ we notice that (a.) was incorrect because x^2-1 factors into $(x+1)(x-1)$. however, it is infact the case that (b.) & (c.) are the same answer! (oops!)

$$\frac{Ax^2+Bx+C}{x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3}$$

and the last three terms also match. For this reason I have to add a point to everyone's score to be fair.

(3.) False, multiply by x^2-1 to get $x^2 = Ax+B \Rightarrow 1=0$ if we equate the coefficient of x^2.

(4.) True, $\frac{x^2}{x^2+3} = 1 - \frac{3}{x^2+3}$ so it works, we make $A=1$ and $B=-3$.

(5) True, $\frac{x+5}{x^2+5x+6} = \frac{x+5}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$

$x+5 = A(x+2) + B(x+3)$

$x=-2$ $\quad 3 = B$

$x=-3$ $\quad 2 = -A$

$\Rightarrow \frac{x+5}{x^2+5x+6} = \frac{-2}{x+3} + \frac{3}{x+2}.$

Remark: The work here wasn't req^d on the quiz, I just include it here to try to justify the logic involved.