Your solutions should be neat, correct and complete. Full credit is not necessarily attained from the correct answer, you can lose points if the solution is not readable. Your solution should be readable to someone who has not read the problem statement. If I have to think about what your calculation means then its not complete. Numerical answers must be given in scientific notation for credit to be awarded. Missing or incorrect units on your answer automatically deducts $\frac{1}{2}$ of the credit. Finally, the answer must be boxed when there is a particular answer to find. Derivation problems are exceptions to this rule.

Recommended Homework from Textbook: problems

1.6, 1.7, 1.31, 1.34, 1.39, 1.43, 1.48, 1.66, 1.72, 1.82, 1.90.

Naturally, I also reccommend you work on understanding whatever details of lecture seem mysterious at first.

Required Reading 1 [1pt] Your signature below indicates you have read:

(a.) I read Lectures 1, 2, 3 and 4 by Cook as announced in Blackboard: ________________.

(b.) I read Chapter 1 of the required text: ________________.

Problem 1 [3pts] Let x, v, M, t be position, speed, mass and time for a particular object. Also, suppose a force $\vec{\mathbf{F}}$ has components described by $\vec{\mathbf{F}} \cdot \hat{\mathbf{x}} = ax$, $\vec{\mathbf{F}} \cdot \hat{\mathbf{y}} = bv^2$ and $\vec{\mathbf{F}} \cdot \hat{\mathbf{z}} = \frac{cM}{t}$. What are the SI-units of the constants a, b, c? You may leave your answer in terms of meters (m), kilograms (kg) and seconds (s).

The units of $\vec{F}\cdot\hat{x}$, $\vec{F}\cdot\hat{y}$ and $\vec{F}\cdot\hat{z}$ are given by the units of $\vec{F}$ as $\hat{x}, \hat{y}, \hat{z}$ are pure, unitless, vectors. Thus,

$$N = [ax] = [a][x] = [a]m \Rightarrow [a] = \frac{N}{m} = \frac{kg\,m/s^2}{m} = \frac{kg}{s^2}$$

$$\therefore \boxed{[a] = kg/s^2}$$

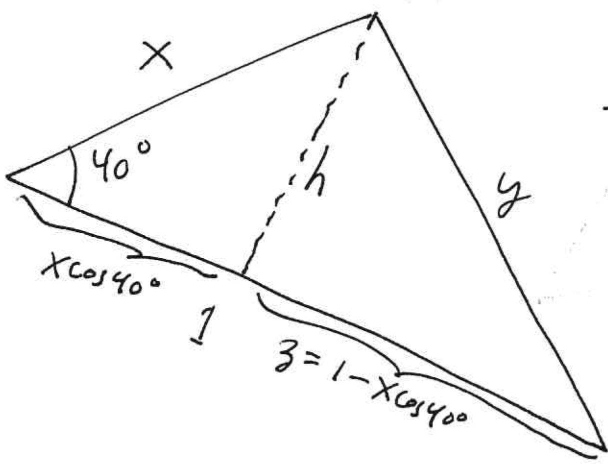
$$N = [bv^2] \Rightarrow [b] = \frac{N}{[v]^2} = \frac{N}{(m/s)^2} = \frac{\frac{kg\,m}{s^2}}{\frac{m^2}{s^2}} = \frac{kg}{m}$$

$$\therefore \boxed{[b] = kg/m}$$

$$N = \left[\frac{cM}{t}\right] \Rightarrow [c] = \frac{N[t]}{[M]} = \frac{\left(kg\frac{m}{s^2}\right)(s)}{kg} = \frac{m}{s}$$

$$\therefore \boxed{[c] = \frac{m}{s}}$$

Problem 2 [3pts] A triangle has area 3 and one of the sides has length 1 with an adjacent angle of 40^o. What are the lengths and remaining angles in this triangle? Please draw a triangle with all the answers clearly written.



$$\frac{1}{2} B h = \text{AREA}$$

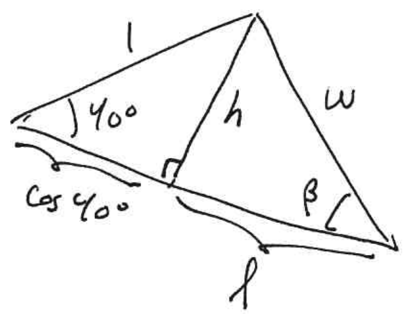
$$\Rightarrow 3 = h/2$$

$$\therefore \underline{h = 6}.$$

$$\sin 40^\circ = \frac{h}{x} \Rightarrow x = \frac{h}{\sin 40^\circ} = \frac{6}{\sin 40^\circ} = \underline{9.334}.$$

$$3 = 1 - x\cos 40^\circ = 1 - 9.334 \cos 40^\circ = \text{negative } \# \ !$$

Thus, my picture needs modification. The other possibility, is the picture below:



$$\frac{1}{2} B h = \text{Area}$$

$$\frac{1}{2}(\cos 40^\circ + \ell) h = 3$$

But, $h = \sin 40^\circ$ hence,

$$\frac{1}{2}(\cos 40^\circ + \ell)\sin 40^\circ = 3$$

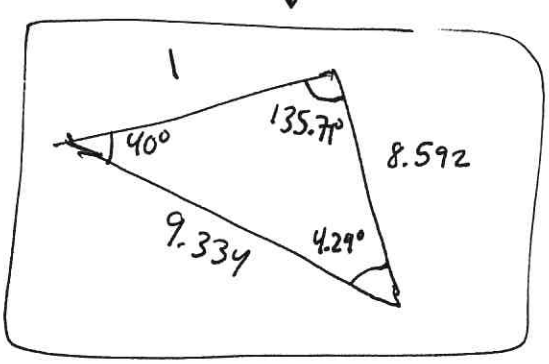
$$\cos 40^\circ + \ell = \frac{6}{\sin 40^\circ}$$

$$\ell = \frac{6}{\sin 40^\circ} - \cos 40^\circ = \underline{8.568}$$

$$\Rightarrow \ell + \cos 40^\circ = 9.334$$

$$w = \sqrt{8.568^2 + \sin^2 40^\circ} = \underline{8.592}$$

$$\beta = \tan^{-1}\left(\frac{h}{\ell}\right) = \tan^{-1}\left(\frac{\sin 40^\circ}{8.568}\right) = 4.29^\circ$$



Problem 3 [3pts] Let $\vec{\mathbf{A}} = \langle 1, 2, 2\rangle$ and $\vec{\mathbf{B}} = \langle 0, -1, 3\rangle$. Find $A, B, \widehat{\mathbf{A}}, \widehat{\mathbf{B}}$ and the angle between the vectors.

$$A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{1+4+4} = \sqrt{9} = \boxed{3. = A}$$

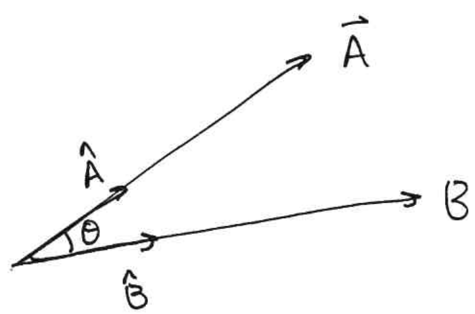
$$B = \sqrt{\vec{B}\cdot\vec{B}} = \sqrt{0+1+9} = \boxed{\sqrt{10} = B}$$

$$\hat{A} = \frac{1}{A}\vec{A} = \boxed{\frac{1}{3}\langle 1, 2, 2\rangle = \langle 1/3, 2/3, 2/3\rangle = \hat{A}}$$

$$\hat{B} = \frac{1}{B}\vec{B} = \boxed{\frac{1}{\sqrt{10}}\langle 0, -1, 3\rangle = \langle 0, -1/\sqrt{10}, 3/\sqrt{10}\rangle = \hat{B}}$$

$$\hat{A}\cdot\hat{B} = \cos\theta \Rightarrow \cos\theta = \frac{1}{3\sqrt{10}}(0-2+6) = \frac{4}{3\sqrt{10}}$$

$$\theta = \cos^{-1}\left(\frac{4}{3\sqrt{10}}\right) = \boxed{65.06^\circ = \theta}$$



See, θ shared by both $\vec{A}$ & $\vec{B}$ and $\hat{A}$ & $\hat{B}$

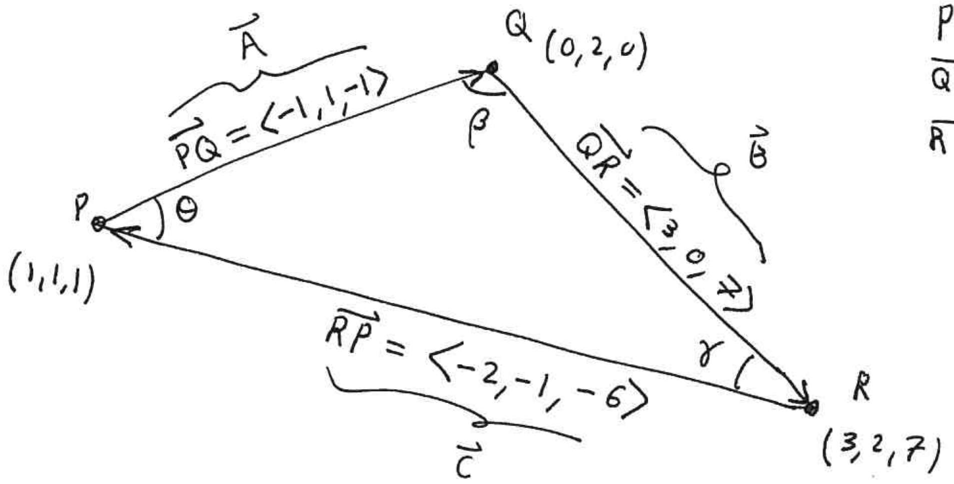
So, I can calculate θ from either

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

or

$$\hat{A}\cdot\hat{B} = \cos\theta.$$

Problem 4 [3pts] A triangle PQR is formed by the triple of points $P = (1,1,1)$ and $Q = (0,2,0)$ and $R = (3,2,7)$. Find the lengths and angles of this triangle. Present your answer as a picture with the sides and angles labeled neatly (you do not need to draw it to scale or perspective, the purpose of the triangle is merely to clearly communicate your conclusions)



$\vec{PQ} = Q - P = \vec{A}$

$\vec{QR} = R - P = \vec{B}$

$\vec{RP} = P - R = \vec{C}$

$A = \| \vec{PQ} \| = \sqrt{3}$

$B = \| \vec{QR} \| = \sqrt{9 + 49} = \sqrt{58}$

$C = \| \vec{RP} \| = \sqrt{4 + 1 + 36} = \sqrt{41}$

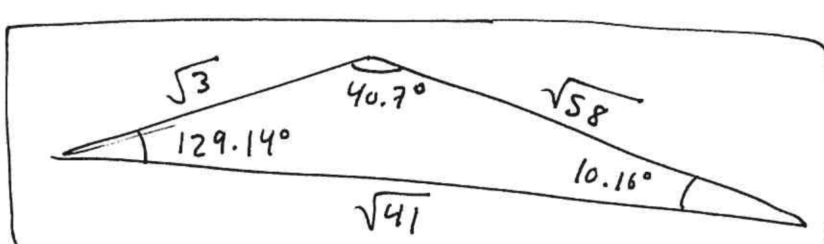
$$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot (-\vec{C})}{AC} \right) = \cos^{-1}\left[\frac{-2+1-6}{\sqrt{3}\sqrt{41}} \right] = \underline{129.14^\circ}.$$

$$\beta = \cos^{-1}\left(\frac{(-\vec{A}) \cdot \vec{B}}{AB} \right) = \cos^{-1}\left[\frac{3+0+7}{\sqrt{3}\sqrt{58}} \right] = \underline{40.70^\circ}.$$

$$\gamma = \cos^{-1}\left(\frac{-\vec{B} \cdot \vec{C}}{BC} \right) = \cos^{-1}\left[\frac{6+0+42}{\sqrt{58}\sqrt{41}} \right] = \underline{10.16^\circ}.$$

Note $\theta + \beta + \gamma \cong 180^\circ$ good check on my work!

Not to scale!



$\sqrt{3} \cong 1.732$

$\sqrt{41} \approx 6.403$

$\sqrt{58} \approx 7.616$

Problem 5 [3pts] A 300.0N force $\vec{\mathbf{F}}_A$ is directed due west. Second, a 200.0N force $\vec{\mathbf{F}}_B$ is directed at a standard angle of 165^o. A third force $\vec{\mathbf{F}}_C$ also acts on the mass in question such that

$$\vec{\mathbf{F}}_A + \vec{\mathbf{F}}_B + \vec{\mathbf{F}}_C = (50.0N)\hat{\mathbf{u}}$$

where the unit-vector is in the direction of the vector $\hat{\mathbf{x}} + 3\hat{\mathbf{y}}$. Calculate the magnitude and standard angle of $\vec{\mathbf{F}}_C$.

$$\vec{F}_A = (300.0N)(-\hat{x}) = \langle -3F_o, 0 \rangle \quad , F_o = 100N$$

$$\vec{F}_B = (200.0N)\,\hat{u}_B = 2F_o \langle \cos 165, \sin 165 \rangle$$

$$\vec{F}_C = \langle F_{cx}, F_{cy} \rangle$$

$$\vec{F}_A + \vec{F}_B + \vec{F}_C = \langle -3F_o, 0 \rangle + \langle 2F_o \cos 165, 2F_o \sin 165 \rangle + \langle F_{cx}, F_{cy} \rangle$$

$$= \langle -3F_o + 2F_o \cos 165 + F_{cx}, \ 2F_o \sin 165 + F_{cy} \rangle$$

Thus, notice $\hat{u} = \frac{1}{\sqrt{1+9}}(\hat{x} + 3\hat{y}) = \langle 1/\sqrt{10}, 3/\sqrt{10} \rangle$ and then,

$$\frac{F_o}{2}\langle 1/\sqrt{10}, 3/\sqrt{10} \rangle = \langle -3F_o + 2F_o \cos 165 + F_{cx}, \ 2F_o \sin 165 + F_{cy} \rangle$$

$$\Rightarrow F_{cx} = F_o\left(\frac{1}{2\sqrt{10}} + 3 - 2\cos 165\right)$$

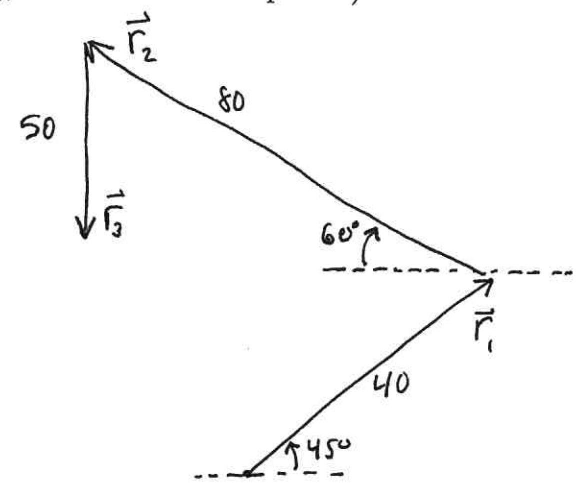
$$F_{cy} = F_o\left(\frac{3}{2\sqrt{10}} - 2\sin 165\right)$$

Thus $F_{cx} = 508.997 = 509.0N$, $F_{cy} = -4.33N$

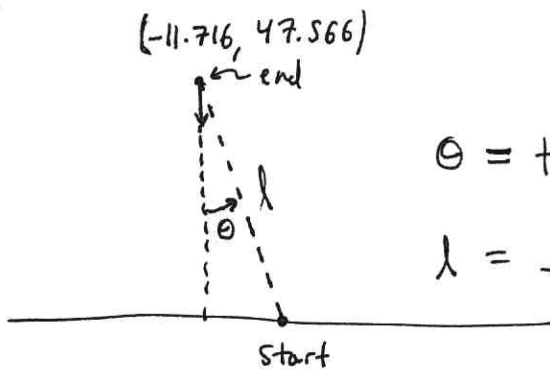
$$F_c = \sqrt{(509.0)^2 + (-4.33)^2} \approx \boxed{509.0\ N}$$

$$\theta = \tan^{-1}\left(\frac{-4.33}{509}\right) = \boxed{-0.48^o}$$

Problem 6 [3pts] A ninja wanders through a dense cloud of hidden mist. He takes 40 steps northeast, then 80 steps 60^o north of west, then 50 steps due south. Assuming he is facing due south at the end, tell him by what angle he should rotate Counter-Clock-Wise(**CCW**) before walking straight to return to his initial starting point. Also, how many steps should will he need to return to the starting point? (answers of the form, hes a ninja so he can just jump, glide, etc whatever, will be amusing, but will not earn points)



	x (steps)	y (steps)
$\vec{r}_1$	$40\cos 45$	$40\sin 45$
$\vec{r}_2$	$-80\cos 60$	$80\sin 60$
$\vec{r}_3$	0	-50
$\vec{r}_1+\vec{r}_2+\vec{r}_3$	-11.716	47.566

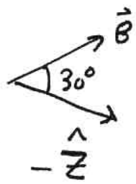


$$\theta = \tan^{-1}\left(\frac{11.716}{47.566}\right) = \boxed{13.84^\circ}$$

$$\ell = \sqrt{11.716^2 + 47.566^2}\ \text{steps} = \boxed{48.99\ \text{steps}}$$

Problem 7 [3pts] Let $\vec{\mathbf{B}}$ be vector of magnitude 30 which makes an angle of 40^o with the positive x-axis and an angle of 30^o with the negative z-axis. Find the explicit Cartesian form of $\vec{\mathbf{B}}$ (if there are many possible answers then parametrize the solution set by an appropriate variable)

$$\vec{B} \cdot \hat{x} = B\cos 40^\circ = 22.98.$$



$$\vec{B} \cdot (-\hat{z}) = -\vec{B} \cdot \hat{z} = B\cos 30^\circ = 25.98$$

Also, Recall from lecture,

$$\vec{B} = (\vec{B} \cdot \hat{x})\hat{x} + (\vec{B} \cdot \hat{y})\hat{y} + (\vec{B} \cdot \hat{z})\hat{z}$$

$$= (22.98)\hat{x} + (\vec{B} \cdot \hat{y})\hat{y} - 25.98\hat{z}$$

Answer, $\vec{B} = 22.98\hat{x} + \lambda\hat{y} - 25.98\hat{z}$ for $\lambda \in \mathbb{R}$

$$\boxed{\vec{B} = \langle 22.98, \lambda, -25.98 \rangle \text{ for } \lambda \in \mathbb{R}}$$

(answer with $\hat{x}, \hat{y}, \hat{z}$ also good, could use other symbol for λ.)

Problem 8 [3pts] Suppose $\vec{r} = \langle x, y, z \rangle$ and $x = 2^t$, $y = t\sin(t^2)$ and $z = \frac{2t}{1+t^4}$. Calculate the following:

(a.) $\frac{d\vec{r}}{dt}$

(b.) $\int \vec{r}\, dt$

(a.) $\vec{r}(t) = \left\langle 2^t, \ t\sin(t^2), \ \frac{2t}{1+t^4} \right\rangle$

$$\frac{d\vec{r}}{dt} = \left\langle \frac{d}{dt}(2^t), \ \frac{d}{dt}(t\sin t^2), \ \frac{d}{dt}\left(\frac{2t}{1+t^4}\right) \right\rangle$$

$$= \boxed{\left\langle \ln(2)2^t, \ \sin t^2 + 2t^2\cos t^2, \ \underbrace{\frac{2(1+t^4) - 2t(4t^3)}{(1+t^4)^2}}_{\text{simplifies to } \frac{2-6t^4}{(1+t^4)^2}.} \right\rangle}$$

ok if left it as

(b.) $\int \vec{r}(t)\,dt = \left\langle \int 2^t dt, \ \int t\sin t^2 dt, \ \int \frac{2t\,dt}{1+t^4} \right\rangle$

$$= \boxed{\left\langle \frac{2^t}{\ln(2)} + C_1, \ \underbrace{\frac{-1}{2}\cos t^2 + C_2}_{\substack{u = t^2 \\ \text{substitution} \\ \text{does it.}}}, \ \underbrace{\tan^{-1}(t^2) + C_3}_{\substack{u = t^2 \\ \text{subst. gives}}} \right\rangle}$$

$$\int \frac{du}{1+u^2} = \tan^{-1}(u) + C_3 = \tan^{-1}(t^2) + C_3.$$

Problem 9 [3pts] Let $\vec{\mathbf{A}}$ be a vector for which $A = 3$ for all time t. Show that $\vec{\mathbf{A}}$ is perpendicular to $\frac{d\vec{\mathbf{A}}}{dt}$ at all times.

$$A = 3 \Rightarrow \vec{A} \cdot \vec{A} = 9$$

$$\Rightarrow \frac{d\vec{A}}{dt} \cdot \vec{A} + \vec{A} \cdot \frac{d\vec{A}}{dt} = 0$$

$$\Rightarrow 2\vec{A} \cdot \frac{d\vec{A}}{dt} = 0$$

$$\Rightarrow \vec{A} \cdot \frac{d\vec{A}}{dt} = 0 \quad \therefore \underline{\vec{A} \perp \frac{d\vec{A}}{dt}}.$$

Problem 10 [3pts] Calculate the projection of $\vec{v} = \langle 1, 1, 1\rangle$ onto the vector $\vec{w} = 2\hat{\mathbf{y}} - \hat{\mathbf{z}}$. Write $\vec{v}$ as a sum of a vector which is colinear to $\vec{w}$ and another vector which is orthogonal to $\vec{w}$.

$$\vec{w} = \langle 0, 2, -1\rangle$$
$$\hat{w} = \frac{1}{\sqrt{5}}\langle 0, 2, -1\rangle$$

$$\begin{aligned}
\mathrm{Proj}_{\vec{w}}(\vec{v}) &= (\vec{v} \cdot \hat{w})\hat{w} \\
&= \left(\langle 1,1,1\rangle \cdot \frac{1}{\sqrt{5}}\langle 0,2,-1\rangle\right)\frac{1}{\sqrt{5}}\langle 0,2,-1\rangle \\
&= \frac{1}{5}(0+2-1)\langle 0,2,-1\rangle \\
&= \boxed{\langle 0, 2/5, -1/5\rangle}
\end{aligned}$$

$$\vec{v} = \langle 0, 2/5, -1/5\rangle + \left(\vec{v} - \langle 0, 2/5, -1/5\rangle\right)$$

$$\boxed{\vec{v} = \underbrace{\langle 0, 2/5, -1/5\rangle}_{\text{colinear to } \vec{w} \text{ by construction.}} + \underbrace{\langle 1, 3/5, 6/5\rangle}_{\mathrm{Orth}_{\vec{w}}(\vec{v})}}$$

You can check
$$\vec{w} \cdot \mathrm{Orth}_{\vec{w}}(\vec{v}) = \frac{6}{5} - \frac{6}{5} = 0.$$