Your solutions should be neat, correct and complete. Correct units must be given on answers and if you are omitting units in calculations then there should be a sentence explaining your custom. Finally, the answer must be boxed where appropriate. For full credit the solution must be written on a print-out of this pdf by hand.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapters 1, 2 of my lecture notes (pdf posted in Canvas)

(b.) Lectures 1, 2, 3, 4, 5, 6, 7, 8, 29 as posted on the course website (see http://www.supermath.info/PhysicsI.html)

Problem 1: (4pts) For each $\vec{\mathbf{A}} = \langle A_x, A_y \rangle$ given below find $\hat{A}$ and the standard angle for $\vec{\mathbf{A}}$.

(a.) $A_x = 3$ and $A_y = 4$,

(b.) $A_x = -8$ and $A_y = 6$

(c.) $A_x = 0$ and $A_y = -11$

(d.) $A_x = -2$ and $A_y = -\sqrt{21}$

Problem 2: (2pts) Suppose $\vec{\mathbf{A}}$ has $A = 10$ directed at standard angle $\theta_A = 30^o$ and $\vec{\mathbf{B}}$ has $B = 20$ directed at standard angle $\theta_B = 225^o$. If $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ then find the magnitude and standard angle of $\vec{\mathbf{C}}$ and illustrate this vector addition with a tip-to-tail diagram.

Problem 3: (10pts) Let $\vec{\mathbf{A}} = 2\hat{\mathbf{x}} - 3\hat{\mathbf{z}}$ and $\vec{\mathbf{B}} = 4\hat{\mathbf{x}} + 5\hat{\mathbf{y}}$.

(a.) calculate $\vec{\mathbf{A}} \bullet \vec{\mathbf{B}}$,

(b.) calculate $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$,

(c.) find $\angle(\vec{\mathbf{A}}, \vec{\mathbf{B}})$,

(d.) find a vector length 7 which is perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

(e.) If $\vec{\mathbf{C}} = \langle \alpha, 1, 2 \rangle$ then find value for α for which the right-handed triple $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ give the edges of a parallel-piped with a volume of 1.

Problem 4: (2pts) Suppose we have three vectors $\vec{A}, \vec{B}, \vec{C}$ all of which are perpendicular to one another; $\angle(\vec{A}, \vec{B}) = 90^o$, $\angle(\vec{A}, \vec{C}) = 90^o$, $\angle(\vec{B}, \vec{C}) = 90^o$. Let $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. Prove that:

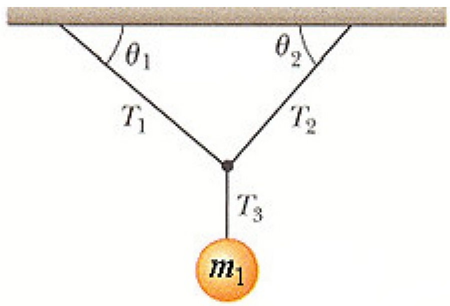
$$D^2 = A^2 + B^2 + C^2$$

where A, B, C denote the magnitudes of $\vec{A}, \vec{B}, \vec{C}$ respectively. Hint: do **not** attempt a picture. Instead, use $D^2 = \vec{D} \bullet \vec{D}$ and the properties of the dot-product to formulate your solution.

Problem 5: (2pts) Suppose $\vec{A} = \langle 1, 2, 2\rangle$ and $\vec{B} = \langle -2, 0, 7\rangle$. Find $\vec{C}, \vec{D}$ such that $\vec{B} = \vec{C} + \vec{D}$ where $\vec{C} \bullet \vec{D} = 0$ and $\vec{C}$ is parallel to $\vec{A}$.

Problem 6: (2pts) A ninja wanders through a dense cloud of hidden mist. He takes 40 steps northeast, then 80 steps 60^o north of west, then 50 steps due south. The ninja then walks 20 steps up the northern face of a vertical tree and steps onto a platform. If the ninja wishes to ambush enemies who wander into his initial position then at what direction (in terms of degrees counter-clockwise from the east-direction) and at what angle below the horizontal should the ninja aim his attacks ? Assume his attacks have laser-like precision and follow a straight line back from his final to his initial position.

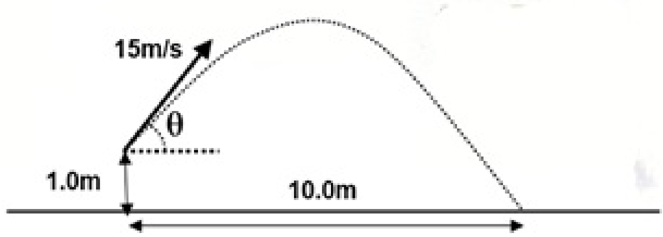
Problem 7: (2pts) Suppose m_1 is suspended by ropes as pictured this makes the tension $T_3 = m_1 g$. Given $\theta_1 = 39^o$ and $\theta_2 = 51^o$, find the values of T_1, T_2 in terms of $m_1 g$. Hint: the sum of the tension forces at the point of attachment must be zero.



Problem 8: (2pts) A projectile is fired in such a way that its horizontal range is equal to four times its maximum height. At what angle of inclination was the projectile fired. Assume a level landscape and ignore air friction.

Problem 9: (3pts) Imagine you shoot an arrow at a speed of v_o at an angle of inclination of $\theta = 30^o$. If the arrow leaves the bow at a height of $2.0\,m$ above the ground and you are trying to shoot a cat (it's evil, in case you're worried) in a tree $200\,m$ away in a branch $22\,m$ above the ground. Find the speed v_o needed in order to shoot the cat.

Problem 10: (3pts) Ron Swanson mistakedly orders a *sausage* sandwhich at a vegan run donut shop. After taking a bite he recoils in horror and throws the faux-meat entree at 15 m/s as shown below. At what θ did Ron Swanson throw the pathetic veggy sandwhich?



Problem 11: (3pts) You shoot a cannon on earth (ignore friction). The cannon ball lands $1000m$ away and you hear the explosion of the cannon ball hitting the level ground some $7.0s$ later than what speed and angle of inclination did you shoot the cannon? You are given that the speed of sound is $333.3m/s$ in the given atmospheric conditions.

Problem 12: (2pts) A ninja on horseback throws a shuriken at horizontally to hit a target $1.25m$ below the release point. If the horse is galloping at $15m/s$ then how far from the target must he throw the shuriken? Assume the ninja can throw a shuriken at $20m/s$ when seated.

Problem 13: (2pts) A rocket car accelerates with a net-force of four times its weight over a distance of L. Then the car applies brakes which give $a = -g$ until the car comes to rest. Find the total distance the car travels. Your answer should be in terms of the given distance L.

Problem 14: (2pts) The force exterted by the wind on the sails of a sailboat is 390 N north. The water exerts a force of 180 N east. If the boat (including its crew) has a mass of 270 kg, what are the magntude and direction of its acceleration ?

Problem 15: (2pts) Let $\widehat{u}$ be a unit-vector directed 30^o north of west. Suppose forces

$$\vec{F}_1 = (3.0\,N)\hat{\mathbf{x}} + (6.0\,N)\hat{\mathbf{y}} \qquad \& \qquad \vec{F}_2 = (20\,N)\widehat{u}$$

act on a body with mass $M = 10\,kg$. Find the magnitude and direction of the resulting acceleration of the mass.

Problem 16: (3pts) Consider coordinate systems (x_1, x_2, x_3) and (y_1, y_2, y_3) and (z_1, z_2, z_3). Suppose a given particle has the following trajectories as measured by the x, y and z observers respective:

$$\begin{aligned}(x_1, x_2, x_3) &= (1-t,\ 2+2t,\ 3-7t+t^2)\\ (y_1, y_2, y_3) &= (3t,\ 4,\ 8t+(t-1)^2)\\ (z_1, z_2, z_3) &= (4-t,\ 4t,\ t^3)\end{aligned}$$

(a.) Calculate velocities with respect to the given coordinate systems: $\vec{v}_X, \vec{v}_Y, \vec{v}_Z$.

(b.) Calculate accelerations with respect to the given coordinate systems: $\vec{a}_X, \vec{a}_Y, \vec{a}_Z$.

(c.) Suppose that (x_1, x_2, x_3) is an inertial coordinate system. Which of the other coordinate systems *could* be inertial as well ?

Remark: if $\vec{a}_Y = R\vec{a}_X$ where R is a rotation matrix then we can calculate, using linear algebra, $\vec{a}_Y \bullet \vec{a}_Y = (R\vec{a}_X) \bullet (R\vec{a}_X) = \vec{a}_X \bullet \vec{a}_X$. Thus, in the case $\vec{a}_Y \neq \vec{a}_X$ we can check if they accelerations are inertially related by checking if they have equal magnitudes.

Problem 17: (3pts) A 55 kg physics student stands on a bathroom scale in an elevator. As the elevator starts moving, the scale reads 45 kg. For the questions below take up to be the positive direction.

(a.) Find the acceleration of the elevator.

(b.) What is the acceleration if the scale reads 67 kg ?

(c.) If the scale reads zero, what is the acceleration of the elevator ?
Should the student worry in this case ?

Problem 18: (2pts) Suppose the acceleration of Bob is given by $\vec{a} = \langle \cos(2t), t, 4e^{2t} \rangle$. If Bob is at $\langle 0, 0, 0 \rangle$ with velocity $\langle 1, 2, 2 \rangle$ at time $t = 0$ then find Bob's position and velocity and speed as functions of time t. (forgive me for omitting units to reduce clutter here)

Problem 19: (2pts) Suppose $\vec{c}$ is a constant vector. Further, suppose the initial position of a particle is $\vec{r}_o$ and the initial velocity is $\vec{v}_o$ at time $t = 0$. Given that the acceleration $\vec{a} = t\vec{c}$, find the velocity and position as a function of time t in terms of the given vectors.

Problem 20: (2pts) Suppose the net-force on a mass M is given by $F = -\beta v$ where v is velocity and $\beta > 0$ is a constant. Let v_o and x_o denote the velocity and position at $t = 0$.

(a.) Calculate the velocity as a function of position x.

(b.) Calculate the velocity as a function of time t.

Hint: for part (a.), assume the motion is one-dimensional, $a = \dfrac{dv}{dt} = \dfrac{dx}{dt}\dfrac{dv}{dx} = v\dfrac{dv}{dx}$.