Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

2.1, 2.8, 2.10, 2.14, 2.23, 2.31, 2.41, 2.47, 2.53, 2.58, 2.77, 2.79, 2.80,

3.8, 3.9, 3.13, 3.22, 3.25, 3.29, 3.31, 3.37, 3.46, 3.51, 3.65, 3.77, 3.84

4.9, 4.10, 4.14, 4.20, 4.29, 4.30, 4.31, 4.43, 4.56, 4.58

I also reccommend you work on understanding whatever details of lecture seem mysterious at first.

Required Reading 2 [1pt] Your signature below indicates you have read:

(a.) I read Lectures 5, 6, 7 and 8 by Cook as announced in Blackboard: ________________.

(b.) I read Chapters 2, 3 and 4 of the required text: ________________.

Problem 11 [3pts] You throw a rock vertically upward. It leaves your hand a distance 1.00m above the ground. When does the ~~rocket~~ rock hit the ground? How far did the rock travel? What is the magnitude of its displacement? You are on Earth and may ignore air friction.

($V_o = 20\,m/s$ was announced 1-25-14)

apparently I've got rockets on the brain.

in m & s

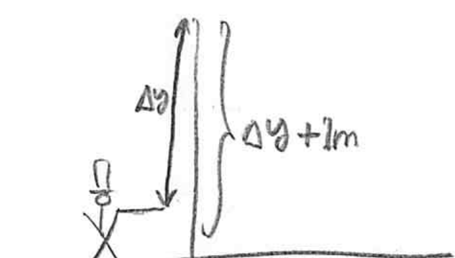
$$y = 1 + 20t - 4.9t^2 \quad \left(y_o + V_o t - \tfrac{1}{2} g t^2\right)$$

$$0 = 1 + 20t - 4.9t^2 \quad \text{when rock hits } y = 0 \text{ ground.}$$

$$\Rightarrow t = \frac{-20 \pm \sqrt{400 + 4(4.9)}}{-2(4.9)} = -0.0494 \text{ or } 4.131$$

$\boxed{\text{The rock hits ground 4.131s after it's thrown.}}$

$$V_f^2 = V_o^2 - 2g\Delta y \Rightarrow \Delta y = \frac{V_o^2}{2g} = \frac{(20\,m/s)^2}{2(9.8\,m/s^2)} = 20.41\,m$$

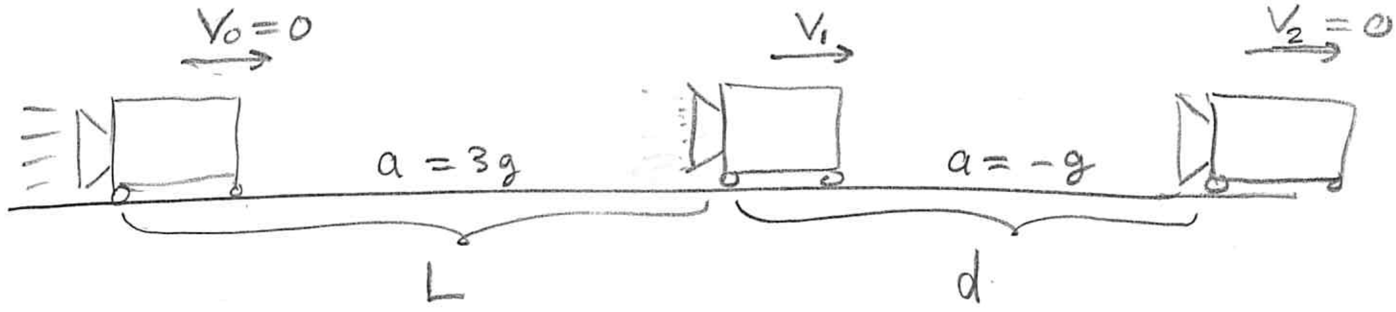


$\Rightarrow$ distance travelled $= 2(20.41m) + 1.0m = \boxed{41.82m}$

Displacement? Well it goes from $y = 1$ to $y = 0$

thus $y(0) - y(1) = -1m \Rightarrow$ magnitude of displacement $= \boxed{1.0m}$

Problem 12 [3pts] A rocket car accelerates at $a = 3g$ over a distance of L. Then the car applies brakes which give $a = -g$ until the car comes to rest. Find the total distance the car travels.



$$V_1^2 = V_0^2 + 2(3g)L \Rightarrow V_1 = \sqrt{6gL}$$

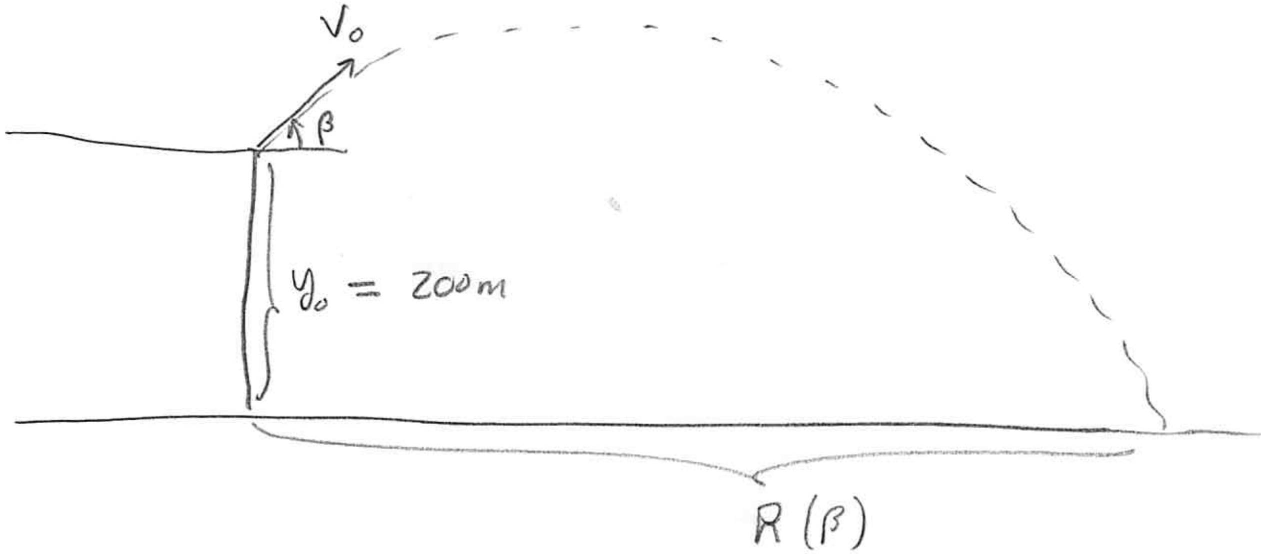
$$V_2^2 = V_1^2 + 2(-g)d \Rightarrow \underbrace{0 = 6gL - 2gd}_{\text{Solve for } d}$$

$$6gL = 2gd$$

$$\underline{d = 3L}\,.$$

total distance $= L + d = \boxed{4L.}$

Problem 13 [3pts] Suppose you launch a water balloon at speed $v_o = 20m/s$ from the top of a $200m$ building at an angle of inclination β. The building is on a level plane on earth and air friciton is neglible. Use calculus to find β which gives the maximum range. You may use some numerical method to solve the algebraic problem which arises in the analysis.



$$y = y_o + (V_o \sin\beta)t - \tfrac{1}{2}gt^2 \rightarrow \underline{y = 200 + 20\sin(\beta)t - 4.9t^2}.$$

Hits ground for $y = 0 = 200 + (20\sin\beta)t - 4.9t^2$

$$\Rightarrow t = \frac{-20\sin\beta - \sqrt{400\sin^2\beta + 4(200)(4.9)}}{-9.8} > 0$$

sol^{n} choose $(-)$ for longer $(+)$ time.

Thus $$t = \frac{20\sin\beta + \sqrt{400\sin^2\beta + 3920}}{9.8}$$

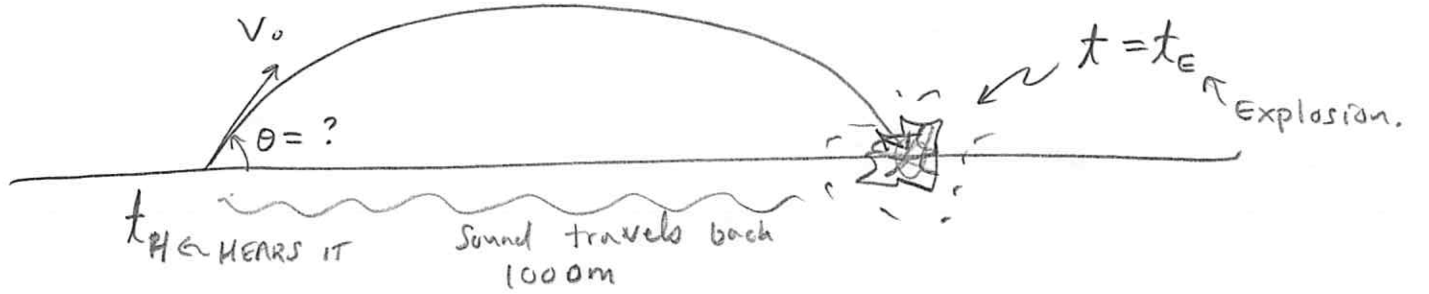
Now, $$R(\beta) = V_{ox}t = V_o\cos\beta\left[\frac{20\sin\beta + \sqrt{400\sin^2\beta+3920}}{9.8}\right]$$

$$\frac{dR}{d\beta} = \frac{V_o}{9.8}\frac{d}{d\beta}\left[\cos\beta\left[20\sin\beta + \sqrt{400\sin^2\beta+3920}\right]\right] \stackrel{set}{=} 0$$ for extrema gets critical β.

Wolfram Alpha yields $\approx$ sol^{n} of $\beta = 0.2954$ radians

$$\Rightarrow \beta = (0.2954)\left(\frac{180^\circ}{\pi}\right) = \boxed{16.93^\circ}$$

Problem 14 [3pts] You shoot a cannon on earth (ignore friction). The cannon ball lands $1000m$ away and you hear the explosion of the cannon ball hitting the level ground some $7.0s$ later than what speed and angle of inclination did you shoot the cannon? You are given that the speed of sound is $333.3m/s$ in the given atmospheric conditions.



We derived $R = \dfrac{V_0^2 \sin(2\theta)}{g}$ given the cannon ball goes from $y=0$ to $y=0$ with no friction. Note,

$$R = V_{0x} t_E = V_0 \cos\theta \, t_E \qquad (a_x = 0 \text{ projectile motion})$$

What else, $R = (333.3\,m/s) t_s$ where t_s = time sound travels.

$$t_H = t_E + t_s = 7$$

Let's see we wish to solve for θ. Eliminate V_0

$$V_0^2 = \frac{gR}{\sin 2\theta} = \left(\frac{R}{\cos\theta \, t_E}\right)^2 \Rightarrow \frac{gR}{\sin 2\theta} = \frac{R^2}{\cos^2\theta \, t_E^2}$$

$$\Rightarrow \frac{g\cancel{R}}{2\sin\theta\cos\theta} = \frac{R^{\cancel{2}}}{\cos^2\theta \, t_E^2}$$

Let's use the other two eqn's:

$$\Rightarrow \frac{\cos^2\theta}{2\sin\theta\cos\theta} = \frac{R}{g t_E^2}$$

$$7 = t_E + t_s$$

$$\Rightarrow \tan\theta = \frac{g t_E^2}{2R}$$

$$R = 333.3 t_s$$

Hence, $t_s = 7 - t_E = \dfrac{R}{333.3} = 3 \Rightarrow t_E = 7 - 3 = 4.$

Therefore, $\tan\theta = \dfrac{(9.8\,m/s^2)(4.0s)^2}{2(1000m)} = 0.0784$

$$\Rightarrow \boxed{\theta = 4.48^\circ} \quad - \; V_0 = \sqrt{\frac{(9.8)(1000)}{\sin(8.96^\circ)}} \frac{m}{s} \Rightarrow \boxed{250.8\,m/s = V_0}$$

Problem 15 [3pts] Suppose $x = R\cos\theta$ and $y = R\sin\theta$ and $z = z_o - \frac{g}{2}t^2$ where θ is a function of time such that $\theta(0) = 0$ and R, z_o, g are constants. Find the position, velocity and accleration vectors. Also, find the displacement vector from time zero to time t.

position: $\underline{\vec{r}(t) = \langle R\cos\theta,\ R\sin\theta,\ z_o - \frac{1}{2}gt^2 \rangle}$. here $\theta = \theta(t)$.

velocity: $\underline{\vec{V}(t) = \frac{d\vec{r}}{dt} = \langle \underbrace{-R\sin\theta \frac{d\theta}{dt},\ R\cos\theta \frac{d\theta}{dt}}_{\text{Chain-Rule}},\ -gt \rangle}$.

acceleration:

$$\boxed{\vec{a} = \frac{d\vec{V}}{dt} = \langle -R\cos\theta\left(\frac{d\theta}{dt}\right)^2 - R\sin\theta\frac{d^2\theta}{dt^2},\ -R\sin\theta\left(\frac{d\theta}{dt}\right)^2 + R\cos\theta\frac{d^2\theta}{dt^2},\ -g \rangle}$$

$$\vec{a} = \langle -R\cos\theta\,\dot{\theta}^2 - R\sin\theta\,\ddot{\theta},\ -R\sin\theta\,\dot{\theta}^2 + R\cos\theta\,\ddot{\theta},\ -g \rangle$$

$$\dot{\theta} = \frac{d\theta}{dt}, \quad \ddot{\theta} = \frac{d^2\theta}{dt^2}$$

Displacement from $t=0$ to $t=t$: $\vec{r}(t) - \vec{r}(0) = \Delta\vec{r}$

$$\Delta\vec{r} = \langle R\cos\theta, R\sin\theta, z_o - \frac{1}{2}gt^2 \rangle - \langle R\cos\theta(0), R\sin\theta(0), z_o \rangle$$

$$\boxed{\Delta\vec{r} = \langle R\cos\theta - R,\ R\sin\theta,\ -\frac{1}{2}gt^2 \rangle}$$

$\theta(0) = 0 \longrightarrow \sin\theta(0) = \sin(0) = 0$.
$\theta(0) = 0 \longrightarrow \cos\theta(0) = \cos(0) = 1$.

Problem 16 [3pts] Suppose $\vec{\mathbf{a}} = 6t\vec{\mathbf{c}}$ where $\vec{\mathbf{c}}$ is a constant vector. Find the position, velocity, and ~~position~~ in terms of the position $\vec{\mathbf{r}}_o$ and velocity $\vec{\mathbf{v}}_o$ at $t = 0$. Also, find an integral which gives the distance travelled.

$$\vec{a} = \frac{d\vec{v}}{dt} = 6t\vec{c} \Rightarrow \int_0^t \frac{d\vec{v}}{d\tau}\,d\tau = \int_0^t 6\tau\vec{c}\,d\tau$$

$$\Rightarrow \vec{v}(t) - \vec{v}(0) = 3t^2\vec{c}$$

$$\Rightarrow \boxed{\vec{v}(t) = \vec{v}_o + 3t^2\vec{c}} \text{ velocity}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_o + 3t^2\vec{c} \Rightarrow \int_0^t \frac{d\vec{r}}{d\tau}\,d\tau = \int_0^t (\vec{v}_o + 3\tau^2\vec{c})\,d\tau$$

$$\Rightarrow \vec{r}(t) - \vec{r}(0) = t\vec{v}_o + t^3\vec{c}$$

$$\Rightarrow \boxed{\vec{r}(t) = \vec{r}_o + t\vec{v}_o + t^3\vec{c}} \text{ position}$$

$$\text{Speed} = \frac{ds}{dt} = \|\vec{v}\| = \sqrt{(\vec{v}_o + 3t^2\vec{c})\cdot(\vec{v}_o + 3t^2\vec{c})}$$

$$\boxed{s(t) = \int_0^t \sqrt{v_o^2 + 9\tau^4 c^2 + 12\tau^2\vec{c}\cdot\vec{v}_o}\,d\tau} \text{ distance travelled}$$

Problem 17 [3pts] A ninja uses a slingshot which allows him to spin explosive clay in a circle of radius $1.25m$. If he is able to spin the slingshot 10 times per second then how far can we shoot the clay? (give it the best angle, assume a level-playing field and no friction)

$$V_0 = \frac{2\pi R}{T}$$ where $T = 0.1s$ and $R = 1.25m$

$$\text{Range} = \frac{V_0^2 \sin(2\theta)}{g}$$; we derived in Lecture for level-ground same y_0 & y_f projectile motion.

$\theta = 45°$ gives maximum range. We can even see this w/o calculus as $-1 \leq \sin(2\theta) \leq 1$ so $\sin(2\theta) = 1$ is the largest and that happens at $\theta = 45°$.

$$\text{Range Max} = \frac{V_0^2}{g}$$

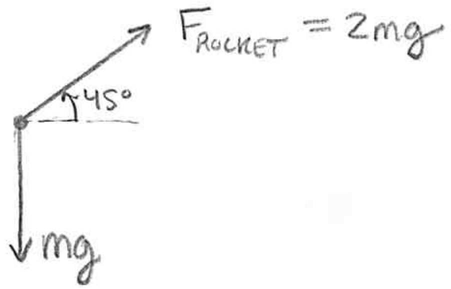
$$= \left(\frac{2\pi R}{T}\right)^2 \frac{1}{g}$$

$$= \frac{4\pi^2 R^2}{gT^2}$$

$$= \frac{4\pi^2 (1.25m)^2}{(9.8\,m/s^2)(0.1s)^2}$$

$$= \boxed{279.8\,m}$$

Problem 18 [3pts] A man has a rocket-boots which produce a thrust of twice his weight. If he turns on his boots and rockets at an angle of 45 degrees from the horizontal for 3 seconds then how high does the man fly ?



$$F_x = (\cos 45°)mg = ma_x \Rightarrow a_x = g/\sqrt{2}$$
$$F_y = (\sin 45°)2mg - mg = ma_y \Rightarrow a_y = (\sqrt{2}-1)g$$

$$y_f = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$$

We can assume $y_0 = 0$ and $V_{oy} = 0$ as the man does not jump etc... it's a fair assumption $V_0 = 0$.

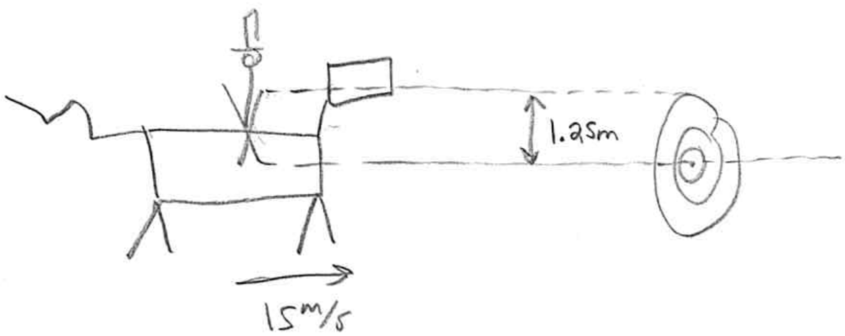
$$y_f = \frac{1}{2}a_y t^2$$
$$= \frac{1}{2}(\sqrt{2}-1)(9.8\,\tfrac{m}{s^2})(3.0s)^2$$
$$= \boxed{18.27m}$$

I've assumed we're on Earth near the surface.

Remark: you could also find the x-displacement.

Problem 19 [3pts] A ninja on horseback throws a shuriken horizontally to hit a target 1.25m below the release point. If the horse is galloping at 15m/s then how far from the target must he throw the shuriken?

at 20 m/s relative the ninja.



$y = 0$ ⇐ a choice, you could set $y=0$ elsewhere. Just be consistent.

$$\begin{pmatrix}\text{velocity of shuriken relative} \\ \text{to the ground}\end{pmatrix} = \left(15\,\tfrac{m}{s}\right) + \left(20\,\tfrac{m}{s}\right)$$

$$\left(\vec{V}_s = \vec{V}_o + \vec{V}_{s'}\right)$$

The equations of motion w.r.t the ground frame, in m, s etc...,

$$X = 35t \qquad (\text{no } x\text{-acceleration})$$

$$y = 1.25 - 4.9t^2 \qquad (\text{horizontal throw} \Rightarrow V_{oy} = 0)$$

$$(g/2 = 4.9\, m/s^2)$$

The time we want to find when it hits target. how $t > 0$ so

Solve $0 = 1.25 - 4.9t^2 \Rightarrow t^2 = \dfrac{1.25}{4.9}$

$$\Rightarrow t = \sqrt{\frac{1.25}{4.9}}\ s = 0.5051 s$$

$$\Rightarrow X = \left(35\,\tfrac{m}{s}\right)(0.5051s)$$

$$\therefore \boxed{X = 17.68m}$$

Problem 20 [3pts] Let $\vec{\mathbf{a}}, \vec{\mathbf{v}}, \vec{\mathbf{r}}$ denote acceleration, velocity and position of a cat in coordinate system S. Let $\vec{\mathbf{a}}', \vec{\mathbf{v}}', \vec{\mathbf{r}}'$ denote acceleration, velocity and position of a cat in coordinate system S'. Furthermore, suppose that $\vec{\mathbf{v}} = \alpha t(\hat{\mathbf{x}} + \hat{\mathbf{z}})$ whereas in another coordinate system $\vec{\mathbf{v}}' = \beta(\hat{\mathbf{x}}' + \hat{\mathbf{y}}')$. Given that α, β are constants and S is an intertial frame of reference, what can we say about the given coordinate system S' ? Do Newton's Laws hold in S'?

Recall, if $\vec{a}_S$ and $\vec{a}_{S'}$ are acceleration measured in the S & S' frames respective then $\vec{a}_S = \vec{a}_{S'}$ for inertially related frames S & S' (this means S & S' are simply related by some straight-line, constant velocity motion and to be entirely honest, possibly a rotation)

So we need only check if the accelerations match.

$$\vec{a}_S = \frac{d\vec{V}_S}{dt} = \frac{d}{dt}\left[\alpha t(\hat{x} + \hat{z})\right] = \alpha(\hat{x} + \hat{z})$$

$$\vec{a}_{S'} = \frac{d\vec{V}_{S'}}{dt} = \frac{d}{dt}\left[\beta(\hat{x}' + \hat{y}')\right] \overset{(*)}{=} 0 \neq \vec{a}_S$$

Thus, the S' coordinate system is <u>non-intertial</u>

(because it was given S was inertial the other frame S' is the guilty party.)

No, Newton's Laws <u>do</u> <u>not</u> hold in S'.

(at least not without the introduction of so-called fictional force terms to account for the frame effects of S')

<u>Remark</u>: I announced $\hat{x}', \hat{y}'$ were <u>not</u> time-dependent to make (*) clear.

Bonus A ninja hound may run a total distance of 51.749 kilometers from where it was summoned by Kakashi. Let us suppose Kakashi gives it instructions to run along a spiral with equations $x = t\cos(t)$,$y = t\sin(t)$ (these are implicitly in kilometers and minutes). How long does Kakashi's hound run before it must return to the dog world from whence it came? Assume the motion starts at $t = 0$. You may use technology to perform the needed integration.

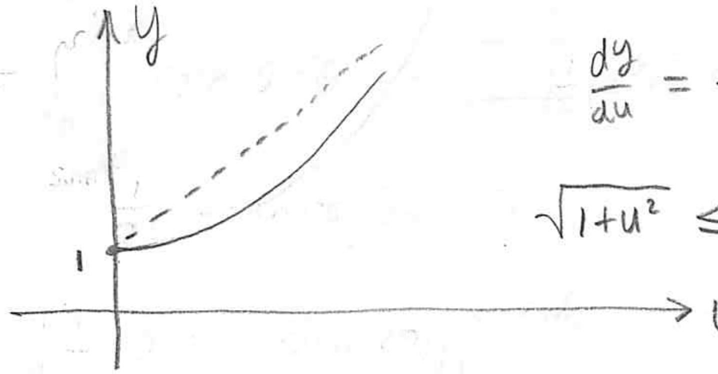
$$V = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}$$

$$= \sqrt{\cos^2 t - \cancel{2t\cos t\sin t} + t^2\sin^2 t + \sin^2 t + \cancel{2t\cos t\sin t} + t^2\cos^2 t}$$

$$= \sqrt{1 + t^2}$$

$$s(t) = \int_0^t \sqrt{1+u^2}\,du$$

Calculate this integral via Wolfram Alpha.



$$\frac{dy}{du} = \frac{2u}{2\sqrt{1+u^2}} = \frac{u}{\sqrt{1+u^2}} < u$$

$$\sqrt{1+u^2} \leq 1 + u$$

$\Rightarrow$ area under $\sqrt{1+u^2}$ from $0 \longrightarrow t$ $< \left(u + \frac{1}{2}u^2\right)\Big|_0^t = t + \frac{1}{2}t^2$

Set $51.749 = t + \frac{1}{2}t^2 \Rightarrow t = 9.22$

So we guess values near (beyond) this to find sol$^{\underline{n}}$ to $s(t) = 51.749$.

$$s(9.5) = \int_0^{9.5} \sqrt{1+u^2}\,du = 46.848$$

$$s(10) = \int_0^{10} \sqrt{1+u^2}\,du = 51.749 \Rightarrow \boxed{\text{Hound runs for 10 minutes}}$$

$$s(10.5) = \int_0^{10.5} \sqrt{1+u^2}\,du = 56.898$$