Physics 231	Mission 3: Momentum, Rotational Physics, Gravity (50+5)pts

Your solutions should be neat, correct and complete. Correct units must be given on answers and if you are omitting units in calculations then there should be a sentence explaining your custom. Finally, the answer must be boxed where appropriate.

Suggested Reading You may find the following helpful resources beyond lecture,

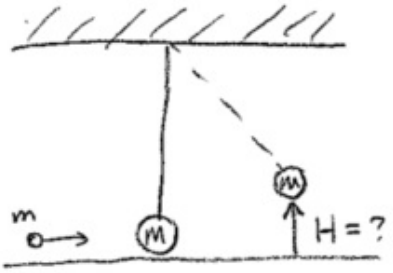
(a.) Lectures 18, 19, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 36 as posted on the course website (see http://www.supermath.info/PhysicsI.html)

(b.) Chapters 5, 6, 7 of my lecture notes (pdf posted in Canvas)

Problem 41: (2pts) Suppose $m_1 = 3.0kg$ is at $\vec{\mathbf{r}}_1 = (1.0m)\langle 1, 2, 3\rangle$ and $m_2 = 4.0kg$ is at $\vec{\mathbf{r}}_2 = (1.0m)\langle -1, 0, 6\rangle$ and $m_3 = 3.0kg$ is at $\vec{\mathbf{r}}_3 = (1.0m)\langle 4, 4, 4\rangle$. Find the center of mass for this system of three masses.

Problem 42: (2pts) An exploding $0.025\,kg$ bullet is fired at 30^o above the horizontal at a speed of $500\,m/s$. At the top of its trajectory is explodes into two equal mass pieces. These pieces fly off in directions which initially form a right angle. How much energy was converted into kinetic energy by the explosion?

Problem 43: (4pts) Suppose a bullet of mass m collides with a pendulum of mass M. If the pendulum swings to a height H then what was the initial speed of the bullet given that:



(a.) the bullet stuck to the pendulum

(b.) the collision was elastic

Problem 44: (3pts) Suppose an ice puck with velocity 15 m/s collides elastically with another identical puck which is at rest. The collision is off-center and thus the collision is not a head-on collision. If the puck initially at rest glides away with speed 10 m/s then what is the speed of the other puck and what angle is found between their paths after the collision?

Problem 45: (6pts) A wheel is given $\alpha = 2.3\ rad/s^2$. If the wheel is initially at rest then find:

(a.) the angular velocity after $2.0\,s$

(b.) the angle through which the wheel turns in the first two seconds

(c.) the torque on the wheel given that $I = 2.0\ kg\,m^2$.

Problem 46: (2pts) If angular acceleration of a self-propelling ninja star is $\alpha = t^4 - 2$ (in rad/s^2) then through what angle in radians does it rotate from time $t = 1$ to $t = 2$ given that at time $t = 1$ the ninja star rotates at $30\, rad/s$.

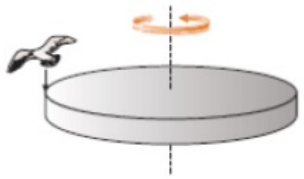
Problem 47: (2pts) A wagon wheel is made with 6 spokes, each of length R and mass M, and a metal rim of radius R and mass $4M$. Suppose the wheel rolls without slipping down an incline of height $h = 100R$. Find the speed v with which the wheel rolls as it reaches the base of the incline.

Problem 48: (2pts) A mass of $m_1 = 100\,kg$ and another of mass $m_2 = 50\,kg$ are attached to a light cable which wraps around a pulley without slipping. Let I be the moment of inertia of the pulley. Starting from rest you observe m_1 fall $4.0\,m$ in a time of $2.0\,s$. What is the value of I ?

Problem 49: (2pts) A yo-yo has $300\,J$ of energy in the form of rotational kinetic energy. The yo-yo also has an angular momentum of $L = 20\,m^2kg/s$. What is the moment of intertia of the yo-yo ?

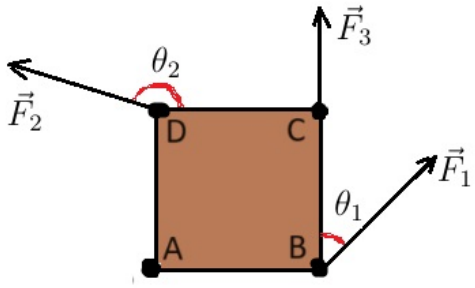
Problem 50: (4pts) Consider a cylindrical turntable whose mass is M and radius is R, turning with an initial angular speed ω_1.

(a.) A parakeet of mass M_p, after hovering in flight above the outer edge of the turntable, gently lands on it and stays in one place on it, as shown above. What is the angular speed of the turntable after the parakeet lands? (Use M,M_b and ω_1 in your answer.)



(b.) Becoming dizzy, the parakeet jumps off (not flies off) with a velocity relative to the turntable. The direction of is tangent to the edge of the turntable and in the direction of its rotation. What will be the angular speed of the turntable afterwards? Express your answer in terms of the two masses M_b and M, the radius R, the parakeet speed $v_{jump} = v_J$ and the initial angular speed ω_1.

Problem 51: (3pts) Suppose $\theta_1 = 30^o$ and $\theta_2 = 160^o$ and the magnitudes of the pictured forces are $F_1 = 20\,N$ and $F_2 = 30\,N$ and $F_3 = 15\,N$. The square of side length $2m$ has mass $M = 20\,kg$ which is uniformly distributed. If the square pivots about the point A then find the net-torque $\vec{\tau}$ on the square. Explain if the direction of the torque is into or out-of the page. Also, find the angular acceleration and indicate its direction (CW or CCW).



Problem 52: (4pts) A mass m has velocity $\vec{v} = \langle bt^2, j\cos\omega t, j\sin\omega t\rangle$ where b, j, ω are constants.

(a.) If the object is initially at $(0,0,0)$ at time $t = 0$ then find $\vec{r}$ at time t,

(b.) Calculate $\vec{r} \times \left(m\frac{d\vec{v}}{dt}\right)$. What have you calculated here ?

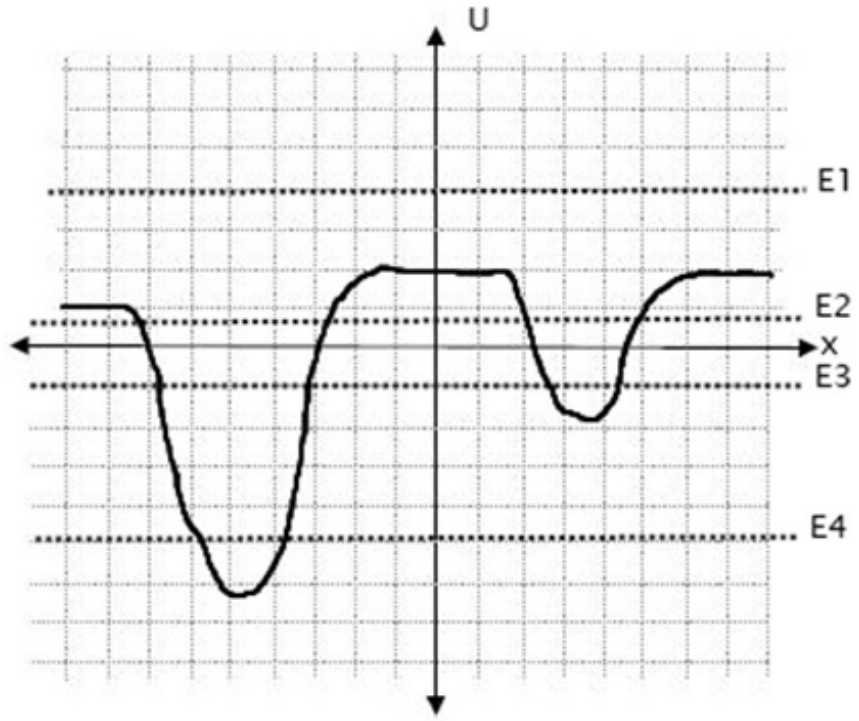
(c.) Calculate $\vec{r} \times (m\vec{v})$. What have you calculated here ?

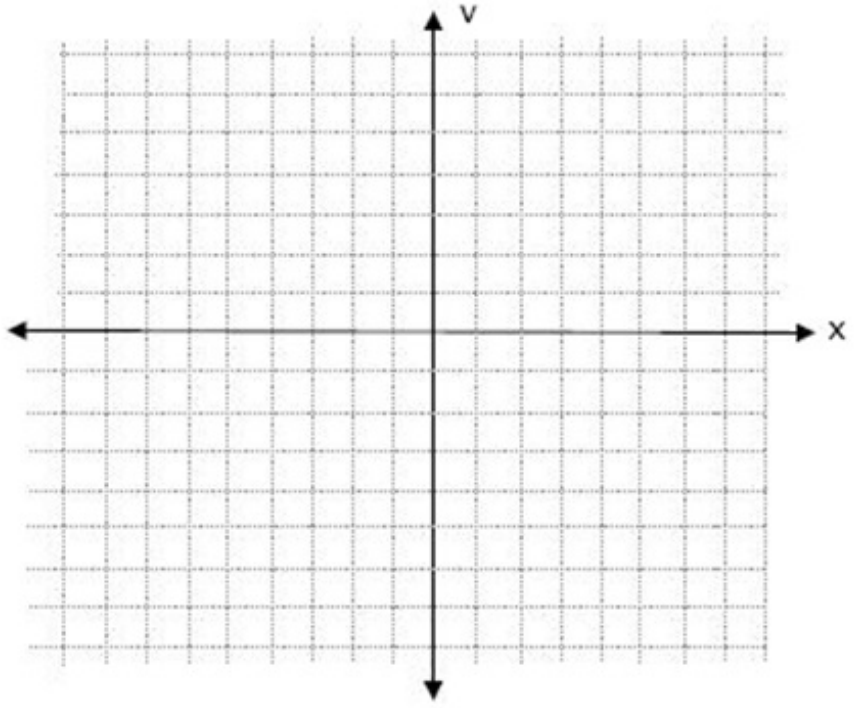
(d.) Calculate $\frac{d}{dt}\left[\vec{r} \times (m\vec{v})\right]$ and compare with part (b.).

Problem 53: (2pts) Show that angular momentum for a mass m orbiting a mass M_S under the force of gravity is conserved. You should assume the origin is at M_S and $M_S >> m$.

Problem 54: (2pts) You measure the gravitational acceleration at a particular altitude is $2m/s^2$. After hiking $10\,km$ vertically you find the gravitational acceleration has dropped to $1\,m/s^2$. Find the mass of this mystery planet and determine your initial distance from the center of the planet.

Problem 55: (5pts) Plot the motions in the xv-plane for total energy E_1, E_2, E_3, E_4 given the potential energy function plotted below. (you should plot the xv – plane beneath the given PE diagram)





Problem 56: (2pts) An ninja concentrates his considerable power and throws his opponent vertically at a speed of 10 km/s. Find the maximum height the opponent reaches. (assume these are earth-based ninjas). What happens if the ninja generates a shadow clone and combines his effort with his clone to throw the opponent vertically at 20 km/s ?

Problem 57: (2pts) A uniform sphere of mass M is located near a thin, uniform rod of mass m and length L. Find the force of gravity on the rod due to the sphere. (this is a calculus problem: break up the rod into infinitesimal masses, find the dF on each dm and integrate!)

Problem 58: (2pts) Let masses $m_1 = 1.0\,kg$ be placed at $(1.0\,m, 0, 3.0\,m)$ and $m_2 = 2.0\,kg$ be placed at $(-1.0\,m, 2.0\,m, 0)$. Find the net gravitational force on $M = 0.030\,kg$ placed at the origin. What is the gravitational acceleration due to m_1 and m_2 at the origin?

Problem 59: (2pts) A planet has mass $M = 3.54 \times 10^{27}\, kg$. A moon orbits the planet in a circular orbit of radius $R = 2.0 \times 10^{8}\, m$. What is the period of the moon's orbit?

Problem 60: (2pts) Four planets of identical mass M orbit in a circular orbit of radius R. The planets are symmetrically placed. Find the speed of their orbit.