Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

6.2, 6.5, 6.11, 6.20, 6.34, 6.47, 6.52, 6.60, 6.71, 6.79, 6.85, 6.93, 6.101

7.5, 7.7, 7.15, 7.35, 7.38, 7.39, 7.47, 7.49, 7.55, 7.64, 7.65, 7.86

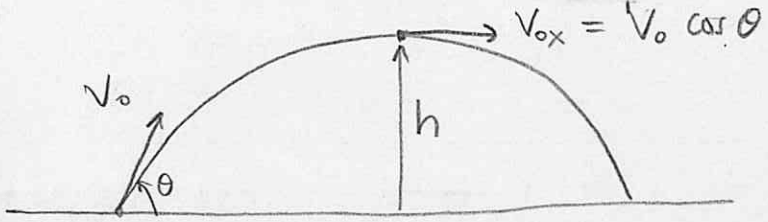
I also reccommend you work on understanding whatever details of lecture seem mysterious at first.

Required Reading 4 [1pt] Your signature below indicates you have read:

(a.) I read Lectures 16, 17, 18 and 19 by Cook as announced in Blackboard: ________________.

(b.) I read Chapter 7 of the required text: ________________.

Problem 31 [3pts] A projectile is shot with a speed v_o at an angle of inclination θ such that it has ~~1/2 as much kinetic energy is half the initial kinetic energy~~ $\frac{1}{2}$ the initial KE when at top of flight. Find θ.



Note: h occurs for $V_y = 0$

$$0 = V_{oy}^2 - 2gh \;, \quad V_{oy} = V_o \sin\theta$$

$$\Rightarrow h = \frac{V_o^2 \sin^2\theta}{2g}$$

$$\frac{1}{2} m V_o^2 = \frac{1}{2}\left(\frac{1}{2} m V_o^2\right) + mgh$$

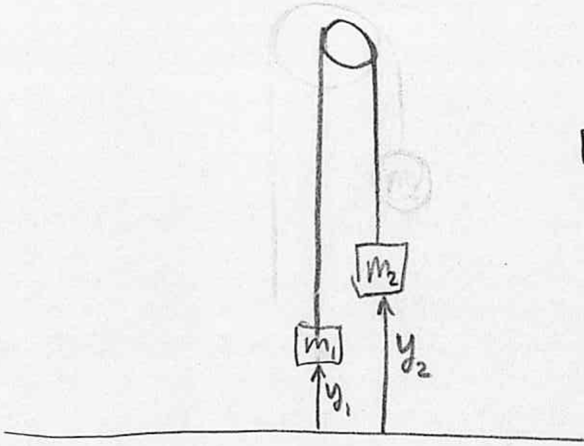
$$\Rightarrow \frac{1}{4} \cancel{m} V_o^2 = \cancel{m}\cancel{g} \frac{V_o^2 \sin^2\theta}{2\cancel{g}}$$

$$\Rightarrow \frac{1}{2} = \sin^2\theta$$

$$\Rightarrow \sin\theta = \frac{\pm 1}{\sqrt{2}} \quad (+ \text{ is sol}^{\underline{n}} \text{ we desire physically speaking})$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \boxed{45^\circ}$$

Problem 32 [3pts] An Atwood machine consists of two masses m_1, m_2 hung over a pulley by a string. Assume the pulley and string are massless and friction is neglible. Suppose $m_2 = 4m_1$. What is the speed of m_2 once it has fallen a distance h from its initial state of rest.



$$E = \underbrace{\tfrac{1}{2}m_1 v^2 + \tfrac{1}{2}m_2 v^2}_{} + m_1 g y_1 + m_2 g y_2$$

share same speed, so can write as $\frac{1}{2}(m_1+m_2)v^2$

Denote $y_{1,0}$, $y_{2,0}$ as initial positions.
Denote $y_{1,f}$, $y_{2,f}$ as final positions.
Apply conservation of energy.

$$m_1 g y_{1,0} + m_2 g y_{2,0} = m_1 g y_{1,f} + m_2 g y_{2,f} + \tfrac{1}{2}(m_1+m_2)V_f^2$$

$$m_1 g (\underbrace{y_{1,0} - y_{1,f}}_{-h}) + m_2 g (\underbrace{y_{2,0} - y_{2,f}}_{h}) = \tfrac{1}{2}(m_1+m_2)V_f^2$$

$h \Leftarrow m_2 > m_1$ so it follows $y_{2,0} > y_{2,f}$.

Recall, $m_2 = 4m_1$ hence,

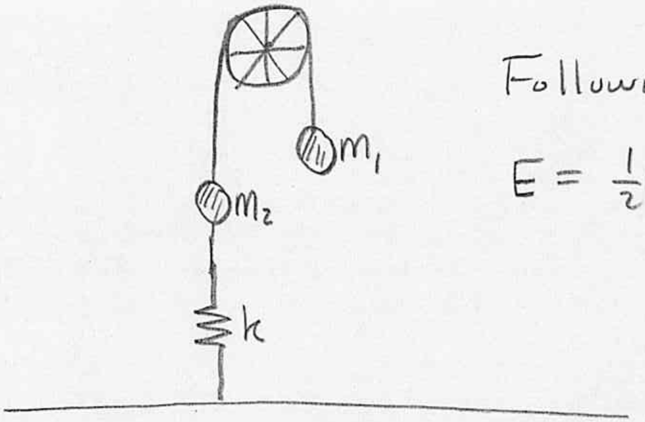
$$-m_1 g h + 4m_1 g h = \tfrac{1}{2}(5m_1)V_f^2$$

$$3gh = \tfrac{5}{2}V_f^2$$

$$V_f^2 = \frac{6gh}{5}$$

$$\therefore \boxed{V_f = \sqrt{\frac{6gh}{5}}}$$

Problem 33 [3pts] Again consider an Atwood machine where the pulley and string are massless and friction is neglible. Assume $m_1 = 10.0kg$ and $m_2 = 3.0kg$. Furthermore, the m_2 is attached to a vertical spring with constant $k = 100N/m$. If the masses have an initial speed of $3.0m/s$ and the spring is at its equilbrium position then find how far the spring stretches. What happens after that point? Describe the motion.



Following notation of last problem,

$$E = \frac{1}{2}(m_1+m_2)V^2 + \underbrace{\frac{1}{2}kx^2}_{} + m_1 g y_1 + m_2 g y_2$$

just added to account for PE of spring. We assume spring is also massless as no mass was given.

Intially, $V_0 = 3\,m/s$, $X_0 = 0$, $y_1 = y_{1,0}$, $y_2 = y_{2,0}$

Finally, $V_f = 0$, $X_f = ?$, $y_1 = y_{1,f}$, $y_2 = y_{2,f}$

$$\frac{1}{2}(m_1+m_2)V_0^2 + m_1 g y_{1,0} + m_2 g y_{2,0} = \frac{1}{2}kx^2 + m_1 g y_{1,f} + m_2 g y_{2,f}$$

$M = 1.0kg$

$$\frac{13M}{2}V_0^2 + 10Mg\underbrace{(y_{1,0} - y_{1,f})}_{h} + 3Mg\underbrace{(y_{2,0} - y_{2,f})}_{-h} = \frac{1}{2}kx^2$$

also $h = x$

$$\frac{13M}{2}V_0^2 + 7Mgh = \frac{1}{2}kh^2$$

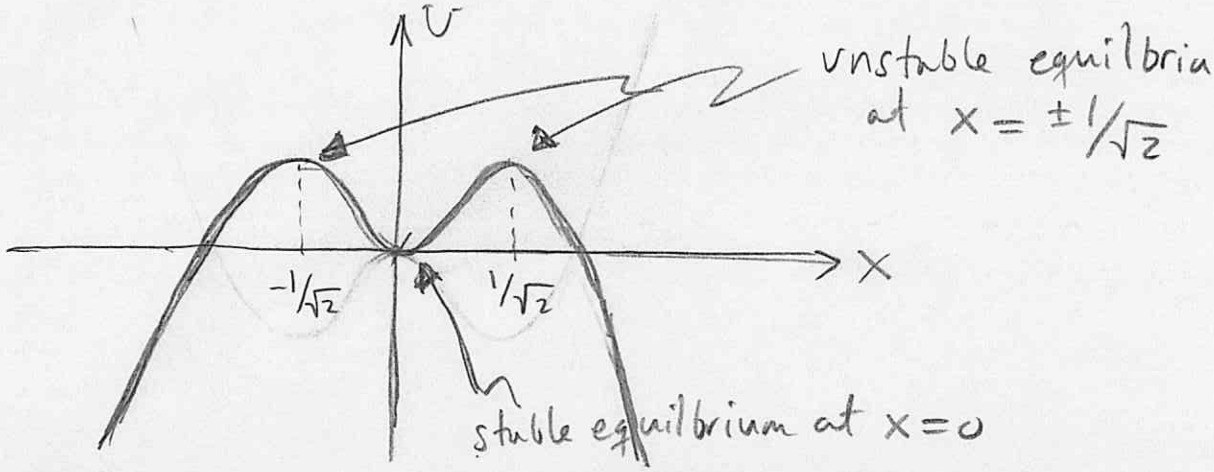
$$\Rightarrow 50h^2 - 68.6h - 58.5 = 0$$

$$\Rightarrow h = \frac{68.6 \pm \sqrt{(68.6)^2 + 4(50)(58.5)}}{100} = -0.5949,\ 1.967.$$

Thus the spring stretches $\boxed{1.967m}$. After that point the spring force $F = -kx = -(100\,\frac{N}{m})(1.967m) = -196.7N$ overcomes the force of gravity mediated by tension to $(m_1 - m_2)g = (7kg)(9.8\,m/s^2) = 68.6N$. Long story short, the system oscillates past this point.

Problem 34 [3pts] Suppose $U(x) = x^2 - x^4$ is the potential energy function. Plot the energy diagram and comment on the stability of any critical points. If F is the force described by this potential energy function then explain where the force is directed right/left. Please give your answer in terms of interval notation. (for example if $2 \leq x \leq 3$ was where F points right then you would say "the force is directed to the right on $[2, 3]$)

$$U(x) = x^2(1-x^2) = x^2(1-x)(1+x)$$



$$\frac{dU}{dx} = 2x - 4x^3 = 2x(1-2x^2) = 2x(1-x\sqrt{2})(1+x\sqrt{2})$$

$$- - - \;|\; +++++ \;|\; ----- \;|\; ++++ \qquad F = -\frac{dU}{dx} = 4x^3 - 2x$$

$$-1/\sqrt{2} \qquad 0 \qquad 1/\sqrt{2}$$

Force points right on $[-1/\sqrt{2}, 0]$ and $[1/\sqrt{2}, \infty)$.

Force points left on $(-\infty, -1/\sqrt{2})$ and $[0, 1/\sqrt{2}]$.

Problem 35 [3pts] Find the potential energy for spring with variable spring "constant". In particular, suppose $F = (k + \alpha x)x$ where k, α are constants. Find the ~~pontential~~ potential energy function for this force.

$$-\frac{dU}{dx} = kx + \alpha x^2$$

$$\frac{dU}{dx} = -kx - \alpha x^2$$

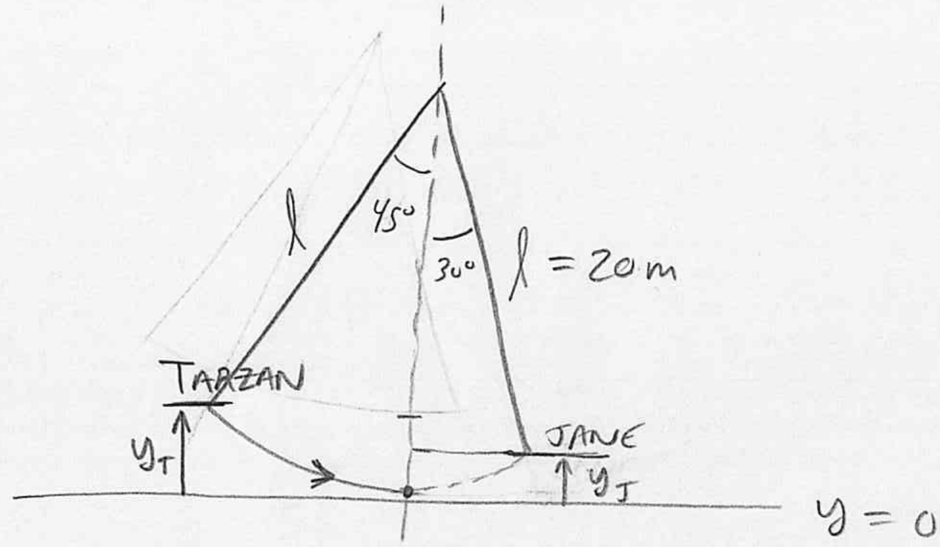
$$\boxed{U(x) = -\frac{1}{2}kx^2 - \frac{1}{3}\alpha x^3}$$

Of course, if we wrote $F = -(k + \alpha x)x$ then we'd find,

$$\underline{U(x) = \frac{1}{2}kx^2 + \frac{1}{3}\alpha x^3}.$$

Both of these suppose $U(0) = 0$ which is a convenience (not a necessity).

Problem 36 [3pts] Problem 7.12 (Tarzan and Jane)



$$y_{TARZAN} = \ell - \ell \cos 45° = 5.858\,m$$

$$y_{JANE} = \ell - \ell \cos 30° = 2.679\,m$$

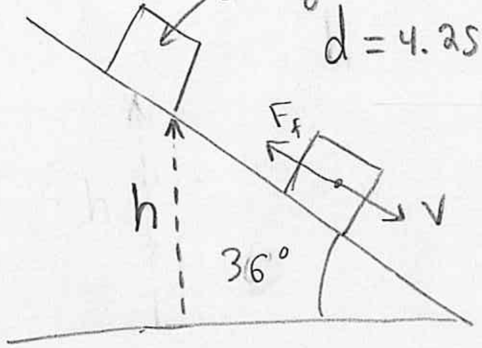
$$\cancel{m}g\,y_T = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g\,y_J$$ ⇐ energy conservation for Tarzan before and just before he hits JANE.

$$2g(y_T - y_J) = v^2$$

$$V = \sqrt{2(9.8)(5.858 - 2.679)}\ \frac{m}{s}$$

$$\boxed{V = 7.894\ m/s}$$

Problem 37 [3pts] Problem 7.32 (Sliding Toolbox)



$mg = 85.0\,N$

$d = 4.25\,m$, $F_f = 22.0\,N$

$\sin 36° = \frac{h}{d} \rightarrow h = (\sin 36°)(4.25\,m)$

$\underline{h = 2.498\,m}$.

$mg = 85.0\,N \rightarrow m = 8.673\,kg$

$$E_f = E_o - F_f d$$

$$\frac{1}{2}mv^2 = mgh - F_f d$$

$$v = \sqrt{\frac{2mgh - 2F_f d}{m}}$$

$$= \sqrt{\frac{2(85)(2.498) - 2(22)(4.25)}{8.673}}\ \frac{m}{s}$$

$$= \boxed{5.235\ m/s}$$

Problem 38 [3pts] Problem 7.36 (deriving force from PE)

$$U(x,y) = \alpha\left(\frac{1}{x^2} + \frac{1}{y^2}\right)$$

$$\vec{F} = -\nabla U = \left\langle -\frac{\partial U}{\partial x},\ -\frac{\partial U}{\partial y},\ -\frac{\partial U}{\partial z} \right\rangle$$

$$= \left\langle \frac{2\alpha}{x^3},\ \frac{2\alpha}{y^3},\ 0 \right\rangle \ \star$$

$$= \boxed{\frac{2\alpha\,\hat{i}}{x^3} + \frac{2\alpha\,\hat{j}}{y^3}}$$

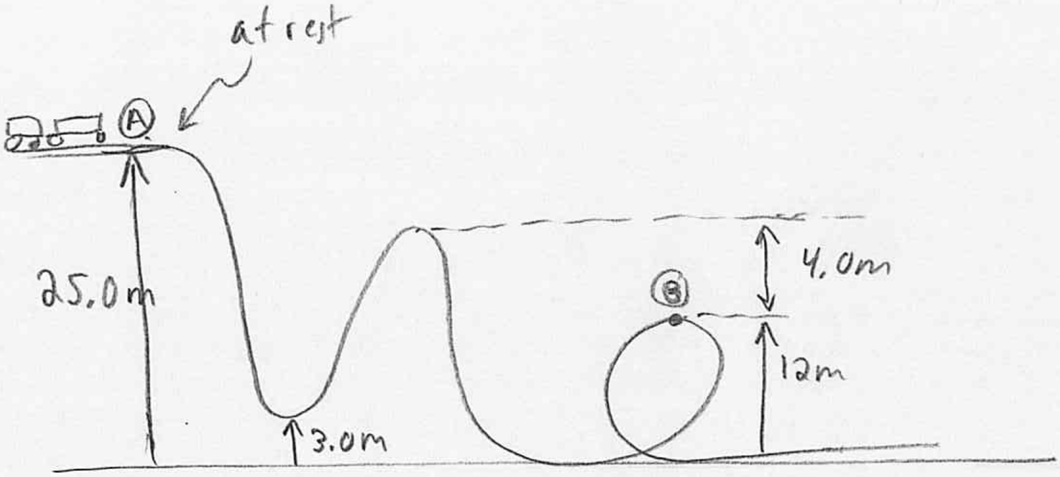
text asked for this,
of course $\star$ is same.

$$\vec{F} = 2\alpha\left(\frac{1}{x^3}\hat{i} + \frac{1}{y^3}\hat{j}\right) \quad \text{also good}$$

or

$$\vec{F} = 2\alpha\left(\frac{\hat{x}}{x^3} + \frac{\hat{y}}{y^3}\right)$$

Problem 39 [3pts] Problem 7.45 (Roller Coaster)



(a.) $V_B = ?$

$$mg\,y_0 = \tfrac{1}{2}mv^2 + mg\,y_B$$

$$v = \sqrt{2g(y_0 - y_B)} = \sqrt{2(9.8)(13)}\ \tfrac{m}{s} = \boxed{15.96\ \tfrac{m}{s}}$$

(b.)

$$\overbrace{n + mg}^{\text{net centripetal force}} = \frac{mv^2}{R}$$

$$n = \frac{mv^2}{R} - mg$$

$$n = m\left(\frac{v^2}{R} - g\right)$$

$$= (350\,kg)\left(\frac{(15.96\,m/s)^2}{6.0\,m} - 9.8\,\frac{m}{s^2}\right)$$

$$= \boxed{1.143 \times 10^4\ N}$$

Problem 40 [3pts] Problem 7.18 (Slingshot)

A slingshot will shoot a
$m = 0.01\,kg$ pebble $h = 22.0\,m$
straight-up.

(a.) How much PE in rubber band of slingshot?

$$PE_{slingshot} = KE_{initially} = PE_{top\ of\ flight}$$
$$= mgh$$
$$= (0.01\,kg)(9.8\,\tfrac{m}{s^2})(22.0\,m)$$
$$= \boxed{2.156\,J}$$

(b.) How high can a mass of $0.025\,kg$ be shot with same slingshot energy?

$$2.156\,J = (0.025\,kg)(9.8\,\tfrac{m}{s^2})\,h_2$$
$$\boxed{h_2 = 8.8\,m}$$

(c.) air friction, mass-dependence of slingshot rubber band, variation of force of gravity implicit within mgh formula. This question is vague...