Your solutions should be neat, correct and complete. Same instructions as Mission 1 apply here.

Recommended Homework from Textbook: problems:

8.3, 8.8, 8.15, 8.16, 8.25, 8.29, 8.43, 8.46, 8.59, 8.73, 8.79, 8.87, 8.100, 8.101, 8.105

I also reccommend you work on understanding whatever details of lecture seem mysterious at first.

Required Reading 5 [1pt] Your signature below indicates you have read:

(a.) I read Lectures 21 and 22 by Cook as announced in Blackboard: ____________.

(b.) I read Chapter 8 and 9 of the required text: ____________.

Problem 41 [3pts] Suppose $m_1 = 3.0kg$ is at $\vec{\mathbf{r}}_1 = (1.0m)\langle 1, 2, 3\rangle$ and $m_2 = 4.0kg$ is at $\vec{\mathbf{r}}_2 = (1.0m)\langle -1, 0, 6\rangle$ and $m_3 = 3.0kg$ is at $\vec{\mathbf{r}}_3 = (1.0m)\langle 4, 4, 4\rangle$. Find the center of mass for this system of three masses.

$$M = m_1 + m_2 + m_3 = 10kg$$

$$\vec{R}_{cm} = \frac{1}{M}\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3\right)$$

$$= \frac{1}{10}\left(3\langle 1,2,3\rangle + 4\langle -1,0,6\rangle + 3\langle 4,4,4\rangle\right)$$

$$= \frac{1}{10}\left(\langle 3-4+12,\ 6+12,\ 9+24+12\rangle\right)(1.0m)$$

$$= \frac{1}{10}\langle 11, 18, 45\rangle(1.0m)$$

$$= \boxed{\langle 1.1m,\ 1.8m,\ 4.5m\rangle}$$

Problem 42 [3pts] Suppose the linear mass density of a cone is given by $\lambda = (3.0\,kg/m^2)x$ for $0 \leq x \leq 30\,cm$ where $x = 0$ corresponds to the tip of the cone and $x = 30\,cm$ gives the base. Find the center of mass for this distribution of mass (notice, while a cone is three-dimensional, clearly the center of mass is on the axis so we are able to treat the problem with single-variate calculus)

$$M = \int_{cone} dm = \int_0^{0.3} 3x\,dx = \frac{3}{2}x^2\Big|_0^{0.3} = \frac{27}{20} = 0.135.$$

$$X_{cm} = \frac{1}{M}\int_{cone} x\,dm$$

$$= \frac{1}{M}\int_0^{0.3} x\,(3x\,dx) \quad : \lambda = \frac{dm}{dx} \hookrightarrow dm = \lambda\,dx$$

$$dm = 3x\,dx$$

in kg, & m

$$= \frac{20}{27}\int_0^{0.3} 3x^2 dx$$

$$= \frac{20}{27}\,x^3\Big|_0^{0.3}$$

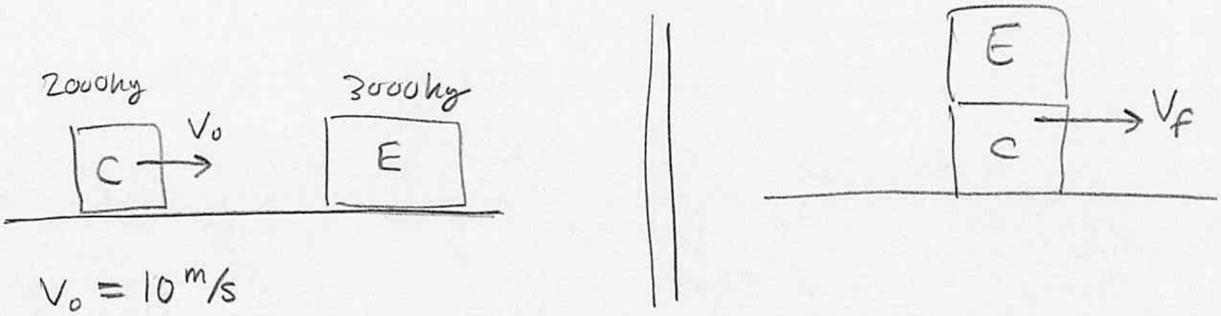
$$= \frac{20}{27}(0.3)^3$$

$$= 0.02\,m$$

$$= \boxed{20\,cm}$$

Hence, $(20cm, 0, 0)$ is center of mass.

Problem 43 [3pts] A 2000 kg car collides with a 3000 kg elephant standing in the intersection. The initial speed of the car is 10 m/s. In the process of the collision the elephant sits on the car. What is the speed of the car-e-phant just after the collision?



$V_0 = 10\,m/s$

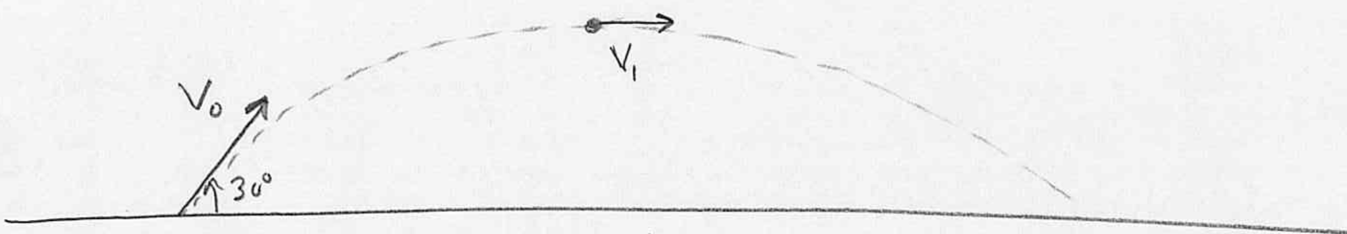
$m = 1000\,kg$

$$2\cancel{m}V_0 = 5\cancel{m}V_f \quad : \text{ Conservation of momentum.}$$

$$V_f = \frac{2}{5}V_0 = (0.4)(10\,m/s) = \boxed{4.0\,m/s}$$

Problem 44 [3pts] An exploding 0.025 kg bullet is fired at 30° above the horizontal at a speed of 500 m/s. At the top of its trajectory is explodes into two equal mass pieces. These pieces fly off in directions which initially form a right angle. How much energy was converted into kinetic energy by the explosion?

$*$



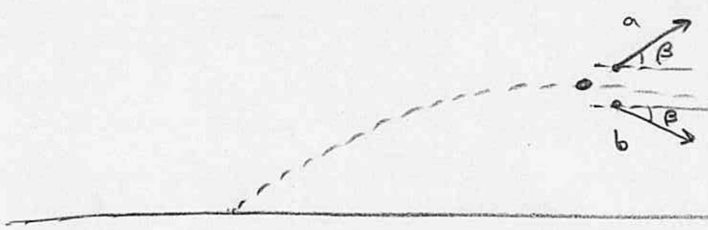
$V_1 = V_0 \cos 30° \quad (a_x = 0)$

Notice, before explosion $\underbrace{\vec{P}_1 = m\vec{V}_1}_{\text{total momentum}}$ then, after the explosion the total momentum is $\vec{P}_a + \vec{P}_b = \frac{m}{2}\vec{V}_a + \frac{m}{2}\vec{V}_b$

Notice, before $\vec{P}_1$ has zero y-component $\Rightarrow V_{ay} = -V_{by}$

thus the picture below is accurate,

$\vec{V}_a = V_a \langle \cos\beta, \sin\beta \rangle$



$\vec{V}_b = V_b \langle \cos\beta, -\sin\beta \rangle$

Conservation of x-momentum yields: (symmetry $\Rightarrow V_a = V_b$)

$$m V_1 = \frac{m}{2} V_a \cos\beta + \frac{m}{2} V_a \cos\beta = m V_a \cos\beta$$

By $*$ we know $\beta = 45° \Rightarrow \underline{V_a = \frac{V_0 \cos 30°}{\cos 45°} = 612.4 \; m/s}$

$$KE_f - KE_1 = \left(\frac{1}{2}\left(\frac{m}{2}\right)V_a^2 + \frac{1}{2}\left(\frac{m}{2}\right)V_a^2\right) - \left(\frac{1}{2} m V_1^2\right)$$

$$= \frac{1}{2} m \left(V_a^2 - V_1^2\right)$$

$$= (0.5)(0.025\,kg)\left[(612.4 \; m/s)^2 - (500 \; m/s)^2\right]$$

$$= \boxed{1562.5 \; J}$$

Problem 45 [3pts] A 3000 kg truck travels past a highway overpass at 20 m/s. A heavy ninja of mass 150kg runs from a bridge which is nearly level with the top of the truck (we can ignore vertical motion). If the truck driver will notice a change of more than 1% in the speed then what is the minimum speed the ninja must run to jump on the truck without being noticed ?

$$P_o = (3000\,kg)(20\,\tfrac{m}{s}) + (150\,kg)V_o = (3150\,kg)\,V_f$$

Worst case $V_f = (0.99)(20\,m/s) = 19.8\,m/s$

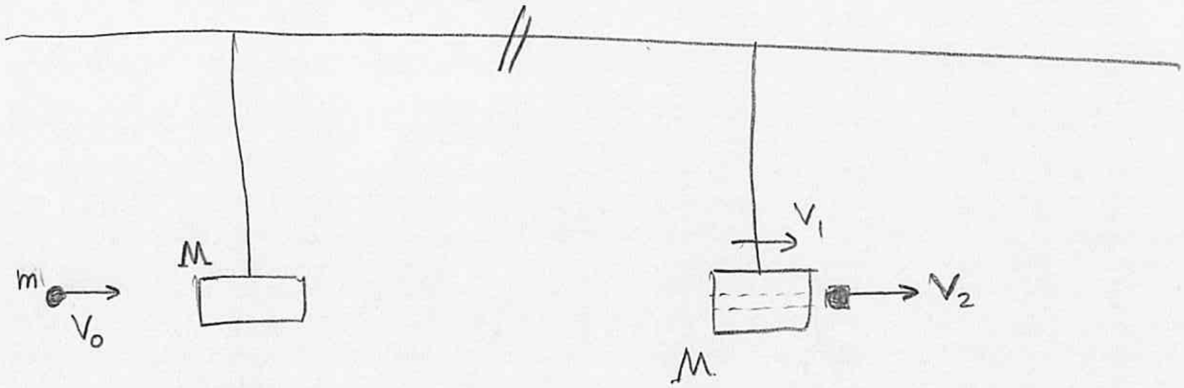
Solve for V_o,

$$V_o = \frac{1}{150\,kg}\left[(3150\,kg)(19.8\,m/s) - (3000\,kg)(20\,m/s)\right]$$

$$\therefore\ V_o = \frac{2370\,kg\,m/s}{150\,kg}.$$

$$\boxed{V_o = 15.8\,m/s}$$

Problem 46 [3pts] A bullet is shot through a clay pendulum bob. In the process of the bullets travel through the pendulum bob it loses half of its kinetic energy. The mass of the pendulum is 0.050 kg. How far does the pendulum swing upward?



$$m V_o = M V_1 + m V_2$$

$$KE_{f,\text{bullet}} = \frac{1}{2} KE_{o,\text{bullet}}$$

$$\frac{1}{2} m V_2^2 = \left(\frac{1}{2} m V_o^2\right)\left(\frac{1}{2}\right) \Rightarrow V_2 = \frac{V_o}{\sqrt{2}}$$

$$m V_o = M V_1 + m \frac{V_o}{\sqrt{2}} \Rightarrow V_1 = \frac{m(1 - 1/\sqrt{2}) V_o}{M}$$

KE of M at base = PE of M at top

$$\frac{1}{2} \cancel{M} V_1^2 = \cancel{M} g h$$

$$h = \frac{V_1^2}{2g}$$

$$\therefore \boxed{h = \frac{1}{2g}\left[\frac{m(1 - 1/\sqrt{2}) V_o}{M}\right]^2}$$

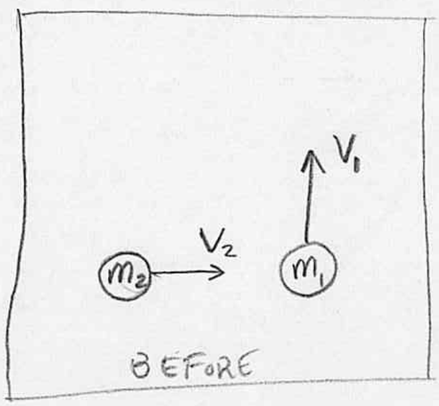
(could put $M = 0.050 kg$, but as m, V_o are not given I'll leave it symbolic)

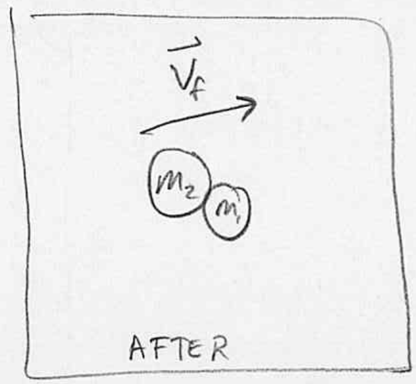
Problem 47 [3pts] Problem 8.37 (football collision)

$m_1 = 110kg$ $\quad$ $m_2 = 85kg$

$V_1 = 8.8\ m/s$

$V_2 = 7.2\ m/s$





$$m_1\vec{V_1} + m_2\vec{V_2} = (m_1+m_2)\vec{V_f}$$: conserve momentum

$$\vec{V_f} = \frac{1}{m_1+m_2}\left\langle (85kg)(7.2\,m/s),\ (110kg)(8.8\,m/s)\right\rangle$$

$$\therefore\ \vec{V_f} = \frac{1}{195kg}\left\langle 612\ kgm/s,\ 968\ kgm/s\right\rangle$$

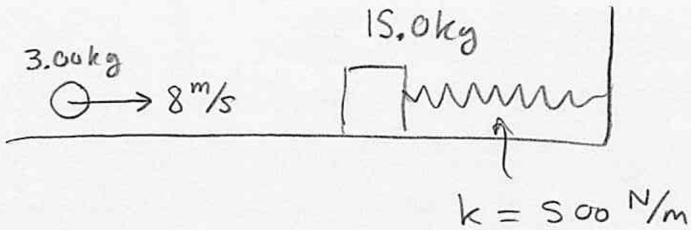
$$\vec{V_f} = \langle 3.138,\ 4.964\rangle \frac{m}{s}$$

$$V_f = \sqrt{(3.138)^2 + (4.964)^2}\ \frac{m}{s} = \boxed{5.873\ m/s = V_f}$$

(magnitude aka speed)

$$\theta = \tan^{-1}\left(\frac{4.964}{3.138}\right) = \boxed{57.7^\circ \text{ (NORTH OF EAST)}}$$

Problem 48 [3pts] Problem 8.44 (spring mass collision)



after collision, stone rebounds at 2.00 m/s to left
Find max. distance block compresses spring after collision.

Mom. Cons. $(3.00\,kg)(8\,\frac{m}{s}) = (3.00\,kg)(-2.00\,\frac{m}{s}) + (15\,kg)(V_0)$

$\Rightarrow \quad 30\,kg\,m/s = 15\,kg\,V_0$

$\Rightarrow \quad V_0 = 2\,m/s$.

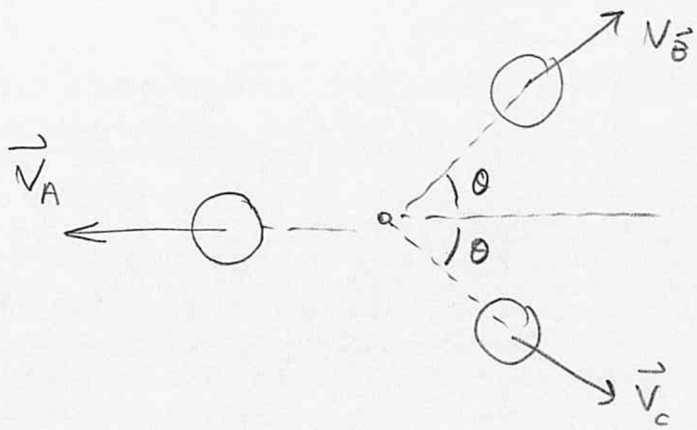
Energy Cons.: $\frac{1}{2} m V_0^2 = \frac{1}{2} k x_{max}^2 \qquad \left(\begin{array}{l} m = 15.0\,kg \\ k = 500\,N/m \end{array} \right)$

$x_{max} = \sqrt{\frac{m V_0^2}{k}}$

$= \sqrt{\frac{(15\,kg)(2\,m/s)^2}{500\,N/m}}$

$= \boxed{0.346\,m}$

Problem 49 [3pts] Problem 8.70 (hockey repulsion)



Given the pucks move away from each other by magnetic force s.t. they have identical speed v at any instant.

$$\vec{P}_A + \vec{P}_B + \vec{P}_c = 0$$ at start and no external force is here to change it.

$$m\vec{V}_A + m\vec{V}_B + m\vec{V}_c = 0$$

$$\Rightarrow \vec{V}_A + \vec{V}_B + \vec{V}_c = 0.$$

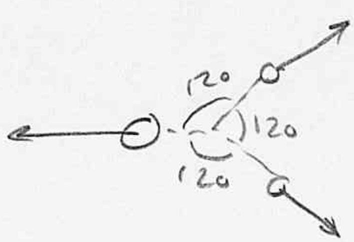
Observe, $V_{By} + V_{cy} = 0$ as $V_{Ay} = 0$ was given.

Thus θ exists as in picture via symmetry.

$$-V + V\cos\theta + V\cos\theta = 0$$

$$2\cos\theta = 1$$

$$\cos\theta = 1/2 \Rightarrow \boxed{\theta = 60^\circ}$$



purely symmetrical each path 120° removed from adjacent path.

Problem 50 [3pts] Problem 8.66 (force, momentum, integral calculus)

young girl $m = 40.0 kg$

initial momentum of $90.0\ kgm/s$ east

Force $F = -(8.20\ \frac{N}{s})t$ (negative as force is directed west)

(a.) for what t_f is $P_f = -60\ \frac{kgm}{s}$. Observe that $P_0 = 90\ \frac{kgm}{s}$ and $F = \frac{dP}{dt}$ hence,

$$\int_0^{t_f} F\,dt = \int_0^{t_f} \frac{dP}{dt}\,dt = P(t_f) - P(0) = P_f - P_0$$

$$P_f - P_0 = -\int_0^{t_f} (8.20)t\,dt$$

$$= -4.1t^2\Big|_0^{t_f}$$

$$= -4.1t_f^2$$

Thus, solve $-60 - 90 = -4.1\,t_f^2 \Rightarrow t_f = \pm\sqrt{\frac{150}{4.1}}\,s$

$$\boxed{t_f = 6.049s}$$

(b.) How much work done on the girl during $0 \le t \le t_f$?

$$W = \Delta KE$$

$$= P_f^2/2m - P_0^2/2m$$

$$= \frac{1}{2(40.0kg)}\left[\left(-60\ \frac{kgm}{s}\right)^2 - \left(90\ \frac{kgm}{s}\right)^2\right]$$

$$= \boxed{-56.25\ J}$$

(c.) $F = ma \Rightarrow |a| = \left|\frac{F}{m}\right| = \left|\frac{(-8.20\ \frac{N}{s})(6.049s)}{40.0kg}\right| = \boxed{1.24\ m/s^2}$

magnitude for 1-dim'l problem ⇒ abs. value.