LECTURE 30:

- We study conservation of angular momentum for systems of particles. This naturally extends the com discussion with linear momentum.

CONSIDER: Suppose $m_1, m_2, \dots, m_N$ are at $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N$ and have external torques $\vec{\tau}_1, \vec{\tau}_2, \dots, \vec{\tau}_N$ applied. Also a force of $\vec{F}_{12}$ is balanced by $\vec{F}_{21}$ between m_1 & m_2 and by the 3rd Law these are equal in magnitude and opposite in direction.

$$\vec{\tau}_{net} = \underbrace{\sum_{j=1}^{N} \tau_j}_{} + \sum_{i,j} \vec{\tau}_{ij}$$

(def$^{=}$) torque of i^{th} particle on itself

$$= \vec{\tau}_{ext} + \sum_{i<j} \vec{\tau}_{ij} + \sum_{i>j} \vec{\tau}_{ij} + \cancel{\overbrace{\sum_{i=j} \vec{\tau}_{ij}}} \rightarrow 0$$

$$= \vec{\tau}_{ext} + \sum_{i<j} \vec{r}_i \times \vec{F}_{ij} + \sum_{i>j} \vec{r}_i \times \vec{F}_{ij}$$

(think)

$$= \vec{\tau}_{ext} + \sum_{i<j} \left(\vec{r}_i \times \vec{F}_{ij} + \vec{r}_i \times \vec{F}_{ji} \right)$$

$$= \vec{\tau}_{ext} + \sum_{i<j} \vec{r}_i \times \cancel{\left(\vec{F}_{ij} + \vec{F}_{ji} \right)} \rightarrow 0 \quad \underline{\text{By 3rd Law}}$$

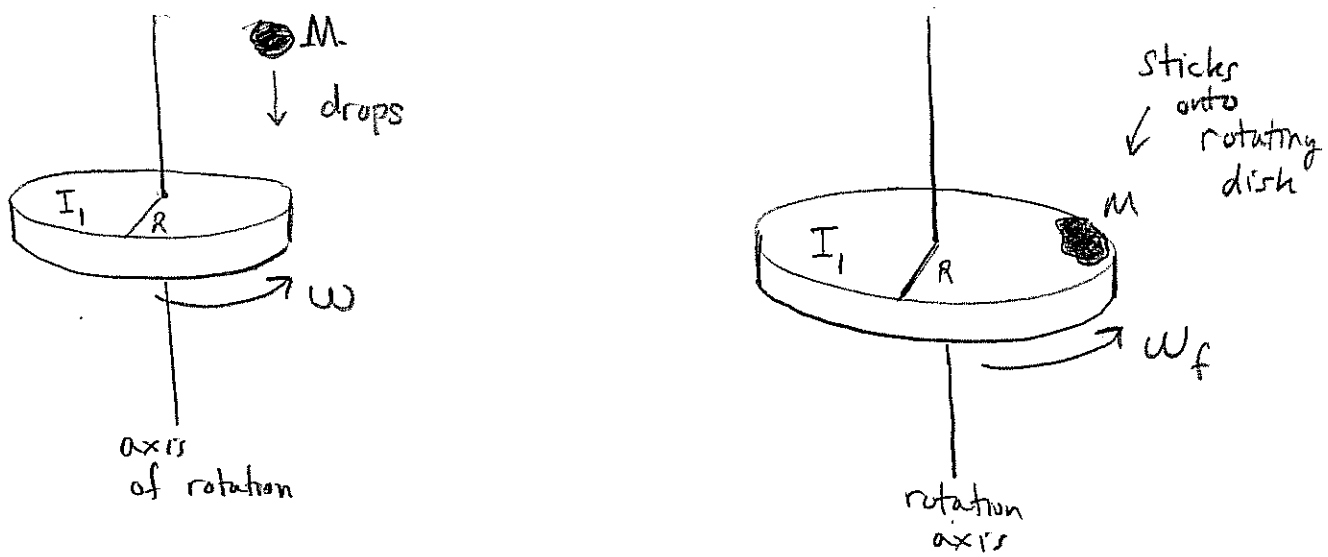
(assuming the internal forces come in pairs, it turns out the magnetic force is not covered by this.)

BOTTOM LINE: $\vec{\tau}_{net} = 0$ makes system isolated and then angular momentum stays constant overall.

Conservation of angular momentum examples

Note: the consituents of the system can change their angular momentum. It's the $\vec{L}_{net}$ that is preserved for $\vec{\tau}_{net} = 0$.

E1



Think of M and I_1 as a system.
Suppose I_1 is fixed vertically. Note gravity is an external force but, it produces no torque on system because it points ∥ to axis.

Hence $\quad \vec{L}_i = \vec{L}_f$

moment of inertia for M at $r=R$.

$$I_1 \omega = I_1 \omega_f + MR^2 \omega_f$$

$$\boxed{\omega_f = \frac{I_1 \omega}{I_1 + MR^2}}$$

(could use $I_1 = \frac{1}{2} M_2 R^2$ if I_1 was a uniform dish of mass M_2)

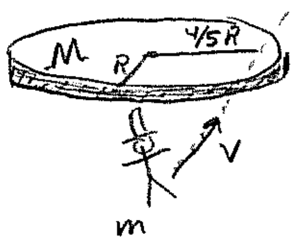
Comment: the example above is inelastic. There was rotational KE lost in collision

$$KE_i = \frac{1}{2} I_1 \omega^2 \neq KE_f = \frac{1}{2}(I_1 + MR^2)\frac{I_1^2 \omega^2}{(I_1 + MR^2)^2}$$

$$\hookrightarrow \frac{KE_f}{KE_i} = \frac{I_1^2}{I_1 (I_1 + MR^2)^2} = \frac{I_1}{I_1 + MR^2} < 1.$$

E2 Suppose a child jumps on a merry-go-round which is a disk of radius R and mass M. If the disk spins horizontally and the child has mass m and jumps onto a point $\frac{4}{5}R$ from the center at tangential velocity v find ω_f for the child / merry-go-round together

We analyze system over an instant of time so torques don't have time to change $\vec{L}$.



$$\vec{L}_i = \vec{L}_f$$

$$\underbrace{\frac{1}{2}MR^2(0)^2}_{\text{for disk}} + \underbrace{m\left(\frac{4R}{5}\right)v}_{\text{for person}} = \left[\frac{1}{2}MR^2 + m\left(\frac{4R}{5}\right)^2\right]\omega_f$$

$$\Rightarrow \boxed{\omega_f = \frac{\frac{4}{5}mRv}{\frac{1}{2}MR^2 + \frac{16R^2}{25}m}}$$

You can make it pretty if you want.

– See Tipler for further examples –

Quantization of Angular Momentum

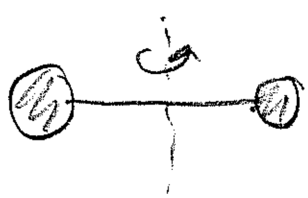
- Angular momentum is <u>quantized</u>

$$L = \sqrt{l(l+1)}\,\hbar \qquad (l = 1, 2, 3, \dots)$$

$$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\,J \cdot s$$

This is extremely small. Compared to usual magnitudes for L we have over 10^{20} quanta in one macroscopic L

- Diatomic Molecules look like little dumbells



← rotational KE which is quantized because this is quantum mechanical problem (small length scale ⇒ QM matters)

$$E = \frac{L^2}{2I} \quad \leftarrow \text{classical formula.}$$

$$= \frac{1}{2I}\, l(l+1)\hbar^2$$

$$= l(l+1)\, E_0 \quad \text{where} \quad \underline{E_0 = \frac{\hbar^2}{2I}}.$$

In contrast to simple harmonic oscillator these levels are not evenly spaced.

- $\vec{J} = \underset{\substack{\text{orbital}\\ \text{ang. mom.}}}{\vec{L}} + \underset{\substack{\text{spin}\\ \text{ang. momentum}}}{\vec{S}}$

$\vec{S}$ is intrinsic to matter. Electrons, Photons etc.. all have a particular type of spin $\vec{S}$.