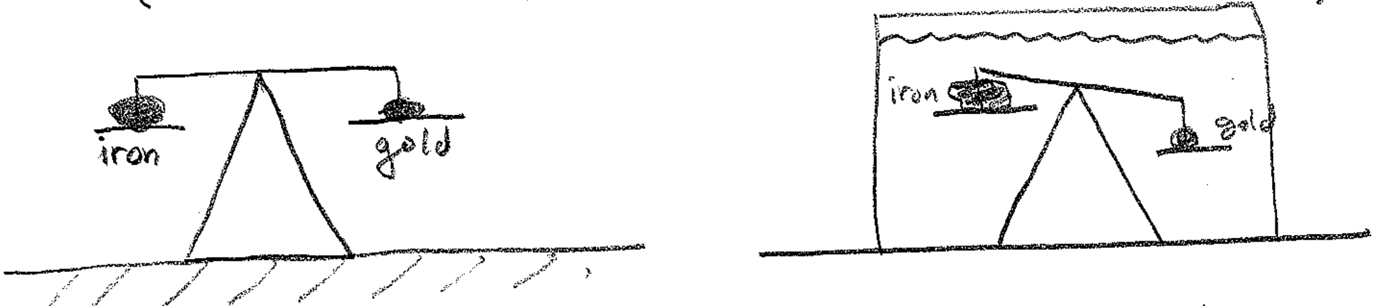
Lecture 35:

- Bernoulli's Eq^{n} and Torricelli's Law. We present the basic eq^{n}'s for fluid flow. To begin we consider the force of buoyancy and Archimede's principle.

<u>Thought Experiment</u>: if we take 10kg of iron and submerge a balance with iron on one side and 10kg of gold on the other then what happens?

(in air balances) (in water reveals materials are different)



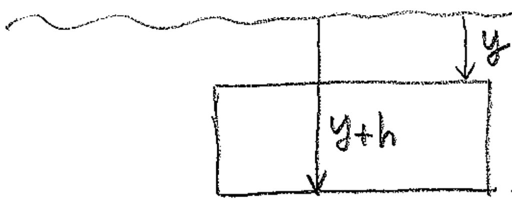
<u>Why?</u> Because the water pushes up with greater force on the iron than the gold. This force is called the buoyant force and the following simple rule allows us to quantify it:

$$F_{buoyancy} = \text{weight of water displaced} = (\rho_{water} V)\, g$$

The iron has $\rho_{iron} = 7960\ \mathrm{kg/m^3}$ whereas $\rho_{gold} = 19{,}300\ \mathrm{kg/m^3}$ thus the iron with mass 10kg takes up a larger volume and hence displaces more water resulting in a greater buoyant force.

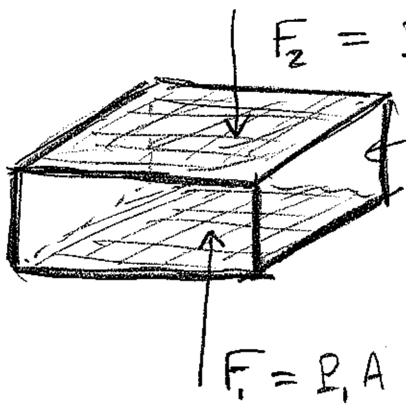
Why is $F_{buoyancy} = M_{water\ displaced}\, g$?

(sideview of submerged block)



$$P_2 = P_0 + \rho g y$$

$$P_1 = P_0 + \rho g (y+h)$$

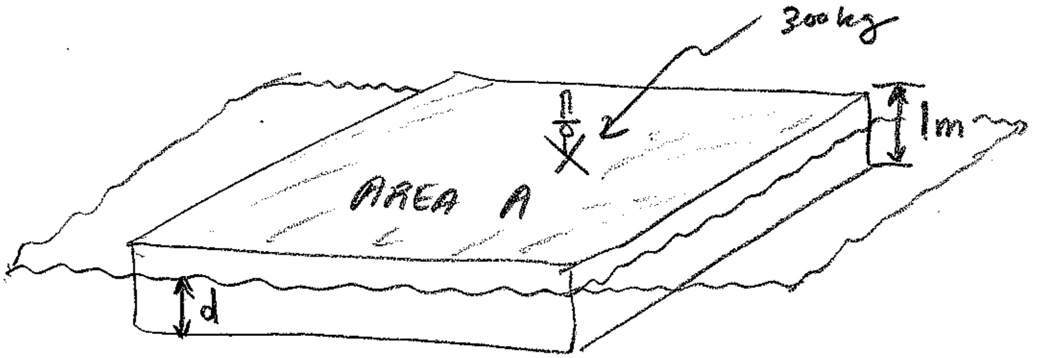


pressure forces on sides cancel out because they're equal and opposite in direction, net force on submerged block is zero (in still water!)

$$F_{net} = F_1 - F_2 = (P_0 + \rho g (y+h))A - (P_0 + \rho g y)A$$
$$= \rho g h A$$
$$= (\rho_{water} V_{displaced})\, g$$
$$= \text{weight of displaced water!}$$

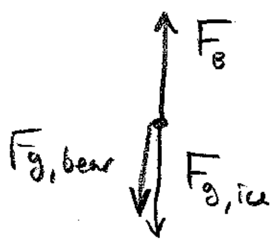
Remark: it's neat this doesn't depend on y.

$\boxed{E1}$ Place polar bear with mass 300kg on block of ice with area of $1000m^2$ and a thickness of 1m. How far is the ice submerged? If we remove the bear then how far is the ice submerged?



$$\rho_{ice} = 920 \, kg/m^3$$

Let's visualize the forces at play here. vertically,



Since $F_B - F_{g,bear} - F_{g,ice} = ma_y$ it follows since $a_y = 0$ that

$$\rho_{water} V_{displaced} \cancel{g} = m_{bear} \cancel{g} + m_{ice} \cancel{g}$$

$$\rho_{water} A d = m_{bear} + \rho_{ice} A (1.0m)$$

$$d = \frac{m_{BEAR} + (920 \, kg/m^3)(1000m^3)(1.0m)}{(1000 \, kg/m^3)(1000m^3)}$$

Thus, $\boxed{d = 0.9203 \, m}$

$$d_{w/o \, bear} = \frac{(920 \, kg/m^3)(1.0m)}{(1000 \, kg/m^3)} = \boxed{0.92 \, m}$$

<u>Remark</u>: for a rectangular block of density ρ_m and height h we can see from $\boxed{E1}$ that

$$d = \frac{\rho_m}{\rho_{water}} h \quad \text{is distance object is submerged.}$$

$$\Rightarrow h - d = \left(1 - \frac{\rho_m}{\rho_{water}}\right) h \quad \text{distance object sticks out of water.}$$

Bernoulli's Eq$^{\underline{n}}$ (see Tipler 440 – 441 for derivation)

Essentially this is energy conservation for smooth flows of liquid at height h above surface of earth, with density ρ and pressure P and velocity V,

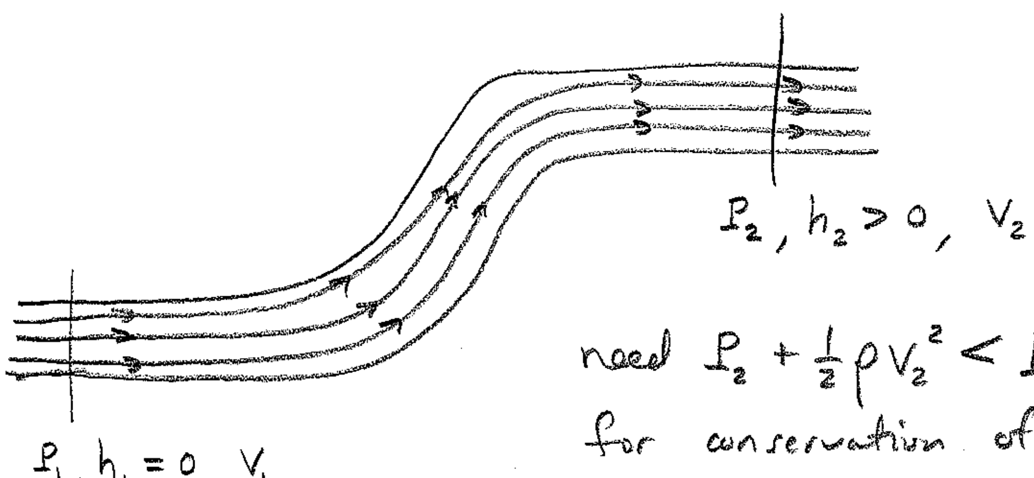
$$\boxed{P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2}$$

random KE

PE due to gravity

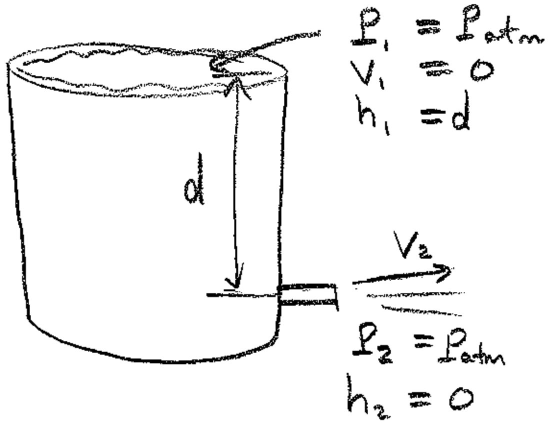
KE associated with group motion of liquid.

(Pressure has to do with the motion of the liquid in all sorts of directions at once)



need $P_2 + \frac{1}{2}\rho V_2^2 < P_1 + \frac{1}{2}\rho V_1^2$ for conservation of energy.

E2



Bernoulli's Eq$^{\underline{n}}$ implies

$$P_{atm} + \rho g d = P_{atm} + \frac{1}{2}\rho V_2^2$$

$$\Rightarrow \boxed{V_2 = \sqrt{2gd}}$$

Torricelli's Law.

Remark: the ρ – independence of this Law is the liquid form of the fact that heavy and light objects share same gravitational acceleration. (do they?)