LECTURE 8

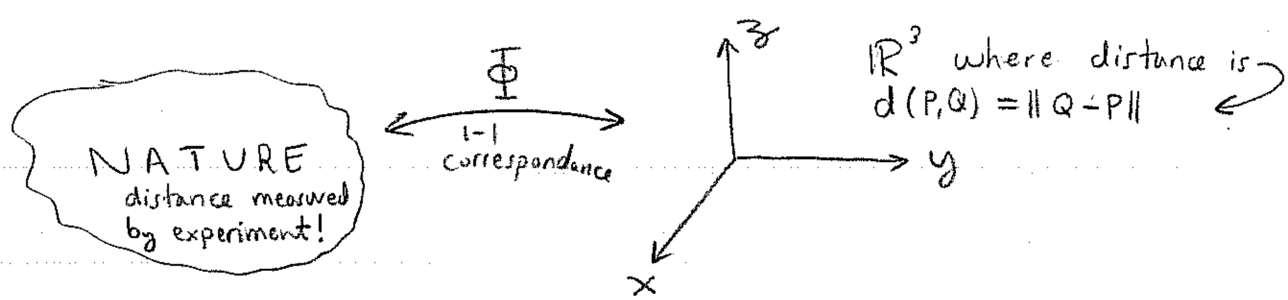
- inertial mass, weight and allowable coordinate systems, NEWTON'S LAWS. Read Tipler for the standard 1^{st} and 2^{nd} Laws. What I present here I learned from my advisor DR. R.O. FULP.

<u>Remark</u>: we have only discussed geometry and motion in LECTURES 1 $\longrightarrow$ 7. Speed, acceleration, position etc... were <u>given</u> but no explaination was yet given for <u>why</u> the motion occurred. Newton's Laws answer this question of <u>why</u>. Newton's Laws state that motion is due to the existence of forces. Forces cause accelerations or a lack thereof causes rest.

<u>AXIOM OF EUCLIDEAN GEOMETRY:</u>

We assume that there is a 1-1 correspondance between the natural world and $\mathbb{R}^3$ with the usual metric structure. Moreover, we assume that there is some method to measure distances and coordinates in the natural world. In short, we assume <u>there exists</u> an <u>intertial reference frame</u>.

<u>Here's a picture of the idea:</u>



This idea is <u>geometry</u>. The physics of geometry is just that we can use mathematical geometry to <u>model</u> physical geometry. The great failure of Kant and others was to identify Nature and $\mathbb{R}^3$. We should remember $\mathbb{R}^3$ is just a <u>model</u>.

Newton's Second Law:

Given a particle, and an inertial frame of reference the acceleration on the particle is in the direction of the net-force $\vec{F}_{net}$ and is inversely proportional to a quantity intrinsic to the particle which we call mass m;

$$\vec{a} = \underset{\text{inertial mass}}{\frac{1}{m}} \vec{F}_{net} \quad , \quad \vec{F}_{net} = \underbrace{\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n}_{\substack{\text{sum of forces,} \\ \text{could be integral} \\ \text{if we have } \infty\text{ly} \\ \text{many forces}}}$$

- Idea: the mass m measures the resistance to a change in velocity ($\frac{d\vec{v}}{dt}$). For a given force, the larger the m the smaller the acceleration.

- Concept: an isolated particle in an inertial frame should stay at constant velocity. This means it either is at rest or in straight-line motion. Notice that if $\vec{F}_{net} = 0$ then

$$\vec{a} = \frac{1}{m}\vec{F}_{net} = 0$$

$$\Rightarrow \frac{d\vec{v}}{dt} = 0 \quad \therefore \quad \boxed{\vec{v}(t) = \vec{v}_0}$$

$$\Rightarrow \frac{d\vec{r}}{dt} = \vec{v}_0 \quad \therefore \quad \boxed{\vec{r}(t) = \vec{r}_0 + t\vec{v}_0}$$

- When net-force is zero the acceleration is zero thus the velocity is constant.

Where do forces come from?

or what forces do we commonly consider

- gravitational force near surface; $\vec{F} = -mg\hat{j}$ (gravitational mass)
- spring force $F = -kx$
- tension force $\vec{T}$ from string pulling
- contact forces (pushing by direct contact)
- electric force
- magnetic force
- friction force

Remark: we assume inertial and gravitational mass are same.

All of the examples above are sources for particular motions of particles. From a fundamental viewpoint these arise from the four fundamental interactions

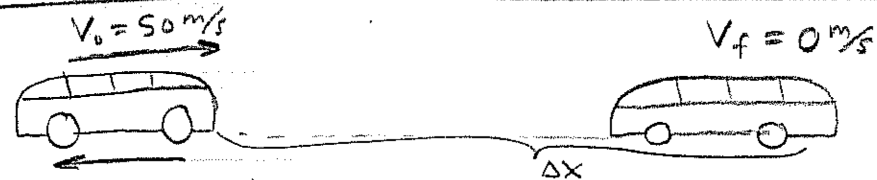
- **GRAVITATIONAL**: mediated by gravitons (massless), long-range.
- **ELECTROMAGNETIC**: mediated by photons (massless), long-range.
- **WEAK**: mediated by $W_{\pm}$, Z bosons (massive) short-range.
- **STRONG**: mediated by gluons at quark-level or mesons at hadronic level, short-range.

All together this collection of models is called the STANDARD MODEL. No explaination is given for the existance of just these forces, or the particles which exist. Except, a great number of consistency arguments make plausibility of the model quite appealing. Mainly, in physics 231/232 we consider gravitational & electromagnetic interactions.

(this is simply an overview, I have more to say at conclusion of this course.)

<u>Remark:</u> we never really explain where the forces come from. Instead, we propose models and seek to explain the maximum # of phenomena with the minimal # of basic causes. Enough philosophy! Let's do some physics!

$\boxed{\text{E1}}$ Suppose a car breaks and stops from it's initial speed of 50 m/s with a friction force which is $\frac{1}{10}$ it's weight. What is the stopping distance



$$F_f = \frac{-1}{10} F_g = F_{net} \quad \text{where } F_g = mg$$

mass of car

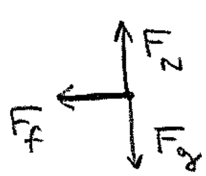
$g = 9.81\,m/s^2$

Thus $ma = \frac{-1}{10} mg$

$\therefore \; a = \frac{-1}{10} g$ ← <u>constant</u> acceleration.

Recall $V_f^2 = V_0^2 + 2a\,\Delta X \Rightarrow \dfrac{-V_0^2}{2a} = \dfrac{-(50\,m/s)^2}{-2g/10} = \Delta X$

<u>Remark:</u> the free-body diagram for a particle helps us to compute $\vec{F}_{net}$ correctly. Note for example above we have:



I noted the vertical components cancelled thus we could focus on ΔX alone.

<u>Remark:</u> A <u>NEWTON</u> is $N = \frac{kg\,m}{s^2}$ this is the force needed to give 1 kg an acceleration of $1\,\frac{m}{s^2}$.

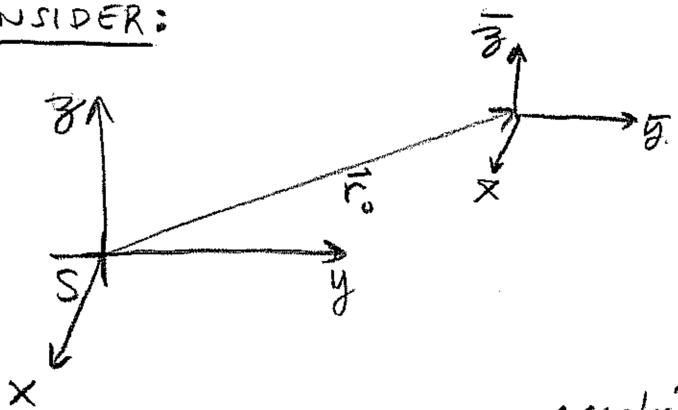
E2 Suppose a particle accelerates at $a_1 = 5\,m/s^2$ when a net force $F = 10N$ applied in the x-direction. What is its mass?

$$a = \frac{F}{m} \Rightarrow m = \frac{F}{a} = \frac{10N}{5\,m/s^2} = \frac{10\,kg\,\cancel{m/s^2}}{5\,\cancel{m/s^2}} = \boxed{2kg}$$

Gravitational vs. Inertial Mass

The formula for weight is $\vec{F}_g = -mg\hat{j}$ and this "m" is the same m as we find in $\vec{a} = \frac{\vec{F}_{net}}{m}$. This means the acceleration given an object from gravity is related to the tendency of the object to resist a change in velocity (inertia).

CONSIDER:



$$\vec{r}_S = \vec{r}_o + \vec{r}_{\bar{S}}$$
$$\vec{V}_S = \vec{V}_o + \vec{V}_{\bar{S}}$$
$$\vec{a}_S = \vec{a}_o + \vec{a}_{\bar{S}}$$

$\frac{d}{dt}$ $\frac{d}{dt}$

acceleration measured by fixed frame S

acceleration of the origin of $\bar{S}$-frame.

acceleration measured in moving frame $\bar{S}$

If we write
$$\vec{a}_S = \frac{\vec{F}_{net}}{m}$$
then what about $\vec{a}_{\bar{S}}$? What is Newton's Law for the moving coordinate system?

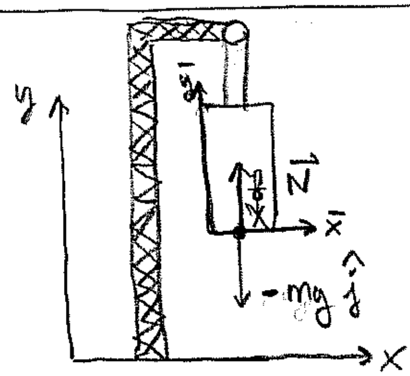
This raises question: what is $\vec{F}_{net}$ in $\bar{S}$?

Continuing:

If we say $\vec{F}_{net}\big|_S = \vec{F}_{net}\big|_{\bar{S}}$ then we cannot say $\vec{a}_S = \dfrac{\vec{F}_{net}}{m} = \vec{a}_{\bar{S}}$ unless we know $\vec{a}_o = \dfrac{d\vec{v}_o}{dt} = \dfrac{d^2\vec{r}_o}{dt^2} = 0$.

A frame $\bar{S}$ with $\vec{a}_o = 0$ is also an inertial frame of reference because Newton's Law holds in the same way as it did in the given, ideal, fixed inertial frame.

E3) ACCELERATED FRAME EXAMPLE: Suppose you are in an elevator which accelerates upward at $1\ m/s^2$. What is the normal force on your feet?



$$m\vec{a} = \vec{N} + \vec{F}_g$$

$$m(1\ m/s^2)\hat{j} = \vec{N} - mg\,\hat{j}$$

$$\vec{N} = mg\,\hat{j} + m(1\tfrac{m}{s^2})\hat{j}$$

$$\boxed{\vec{N} = m(g + 1\tfrac{m}{s^2})\,\hat{j}}$$

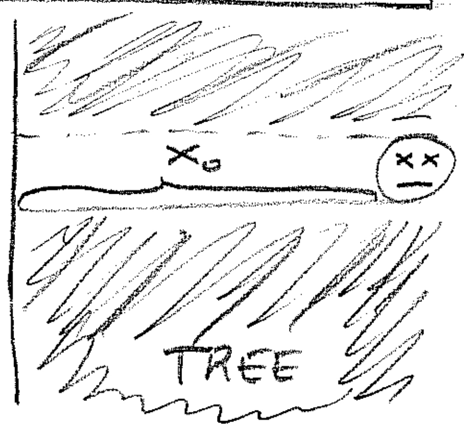
Your effective weight is larger than your standard weight on earth of mg.
From the perspective of the elevator frame $\bar{S}$

$$m\vec{a}_{\bar{S}} = 0 = \vec{N} - mg\,\hat{j} - \underbrace{m(1\tfrac{m}{s^2})\hat{j}}$$

"ficticious" forces have coefficient of m. Is mg ficticious? ... General Relativity.

Remark: we mostly work with non-accelerating frames, but I included E3 here to make you think more about the context in which $\vec{F}_{net} = m\vec{a}$ applies.

E4 A squirrel with initial speed V_0 goes a distance X_0 into a tree. Suppose the squirrel decellerates uniformly. What was the braking force applied on the squirrel from the tree? Squirrel mass is m_0



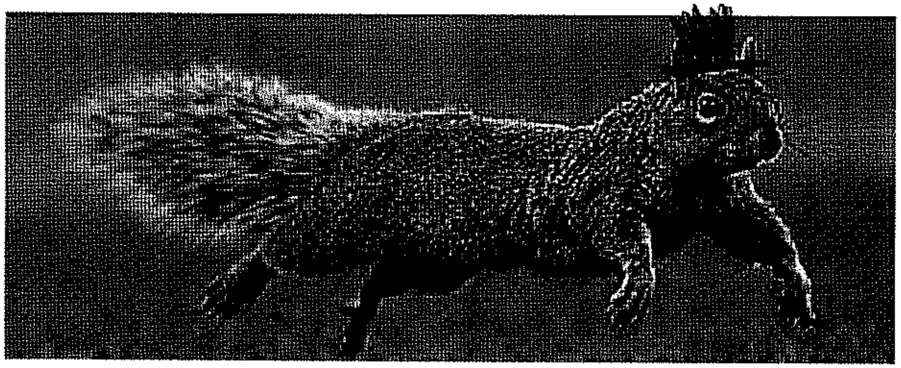
We can find the acceleration from

$$V_f^2 = V_0^2 + 2a\Delta X$$

$$\Rightarrow 0 = V_0^2 + 2aX_0 \quad \therefore \quad a = \frac{-V_0^2}{2X_0}$$

$$F_{net} = m_0 a = \underbrace{\boxed{\frac{-m_0 V_0^2}{2X_0}}}_{}$$

negative since it points left.



1.00×10^{-3} kg

1290 m/s

5 cm