SCALAR SUPERFIELD: U

Def^{n}/ A scalar superfield is a function from superspace to $\mathbb{C}$; $(x^m, \theta^\alpha, \bar{\theta}_{\dot\alpha}) \longmapsto c \in \mathbb{C}$. The standard notation:

$$U(x,\theta,\bar\theta) = f + \theta\phi + \bar\theta\bar\chi + \theta\theta m + \bar\theta\bar\theta n + \theta\sigma^m\bar\theta v_m + \theta\theta\bar\theta\bar\lambda + \bar\theta\bar\theta\theta\psi + \theta\theta\bar\theta\bar\theta d(x)$$

$\{f, \phi_\alpha, \bar\chi^{\dot\alpha}, m, n, v_m, \bar\lambda^{\dot\alpha}, \psi_\alpha, d\}$ are functions from $\mathbb{R}^4 \to \mathbb{C}$ (component fields)

This is the component field expansion. Notice $n_{Boson} = n_{ferm.}$

Bosons: $f, m, n, v_m, d \longrightarrow 1+1+1+4+1 = 8$ dof.
Fermions: $\phi, \bar\chi, \bar\lambda, \psi \longrightarrow 2+2+2+2 = 8$ dof.

Physically the component fields are just ordinary quantum fields (carry a representation of Poincaire). What is extra here is that the superfield U has grouped bosonic and fermionic fields into a common "multiplet". We will see next how SUSY transforms one component field to another.

$$\delta_\epsilon U = \delta_\epsilon f + \theta\delta_\epsilon\phi + \bar\theta\delta_\epsilon\bar\chi + \theta\theta\delta_\epsilon m + \bar\theta\bar\theta\delta_\epsilon n + \theta\sigma^m\bar\theta\delta_\epsilon v_m + \theta\theta\bar\theta\delta_\epsilon\bar\lambda + \bar\theta\bar\theta\theta\delta_\epsilon\psi + \theta\theta\bar\theta\bar\theta\delta_\epsilon d$$

$$\delta_\epsilon U \equiv (\epsilon Q + \bar\epsilon\bar Q)U(x,\theta,\bar\theta)$$

expanded a top of page.

$$\boxed{Q_\alpha = \partial_\alpha - i\sigma^m_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_m} \qquad \boxed{\bar Q^{\dot\alpha} = \bar\partial^{\dot\alpha} + i\theta^\beta\sigma^{m\dot\alpha}_\beta\partial_m}$$

By matching up powers of $\theta, \bar\theta, \theta\theta, \bar\theta\bar\theta, \ldots$ we can determine how δ_ϵ transforms the components.

(I think δ_ϵ is really a derivation, a tangent vector at the identity of the super Lie Group which acts on functions of the space which the group acts, namely superspace.)

Component Field Variations

So equating $\delta_\epsilon(f+\theta\phi+\bar\theta\bar\chi+\cdots+\theta\theta\bar\theta\bar\theta d)$ with $\delta_\epsilon f+\theta\delta_\epsilon\phi+\cdots$ yields after some tribulation,

$$
\begin{aligned}
\delta f &= \epsilon\phi+\bar\epsilon\bar\chi\\
\delta\phi &= 2\epsilon m+\sigma^m\bar\epsilon(i\partial_m f+V_m)\\
\delta\bar\chi &= 2\bar\epsilon n+\epsilon\sigma^m(i\partial_m f-V_m)\\
\delta m &= \bar\epsilon\bar\lambda-\tfrac{i}{2}\partial_m\phi\sigma^m\bar\epsilon\\
\delta n &= \epsilon\psi+\tfrac{i}{2}\epsilon\sigma^m\partial_m\bar\chi\\
\delta V_m &= \epsilon\sigma_m\bar\lambda+\psi\sigma_m\bar\epsilon+\tfrac{i}{2}\epsilon\partial_m\phi-\tfrac{i}{2}\partial_m\bar\chi\bar\epsilon\\
\delta\bar\lambda &= 2\bar\epsilon d+\tfrac{i}{2}\bar\epsilon\partial^m V_m+i\epsilon\sigma^m\partial_m m\\
\delta\psi &= 2\epsilon d-\tfrac{i}{2}\epsilon\partial^m V_m+i\sigma^m\bar\epsilon\partial_m n\\
\delta d &= \tfrac{i}{2}\partial_m\left[\psi\sigma^m\bar\epsilon+\epsilon\sigma^m\bar\lambda\right] \longleftarrow \text{total derivative important later.}
\end{aligned}
$$

Lets see how the δf transformation is calculated, we want to keep all terms with no θ's in $\delta_\epsilon U$,

$$\epsilon QU=\epsilon^\alpha\left(\frac{\partial}{\partial\theta^\alpha}-i\sigma^{m\alpha}_{\dot\rho\alpha}\bar\theta^{\dot\rho}\partial_m\right)(f+\theta\phi+\bar\theta\bar\chi+\cdots)\cong\epsilon^\alpha\frac{\partial}{\partial\theta^\alpha}\theta^\rho\phi_\rho=\epsilon\phi$$

$$\bar\epsilon\bar QU=\bar\epsilon_{\dot\alpha}\left(\frac{\partial}{\partial\bar\theta^{\dot\alpha}}+i\theta^\rho\sigma_\rho^{m\dot\alpha}\partial_m\right)(f+\theta\phi+\bar\theta\bar\chi+\cdots)\cong\bar\epsilon_{\dot\alpha}\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta_{\dot\rho}\bar\chi^{\dot\rho}=\bar\epsilon\bar\chi$$

$$\therefore(\epsilon Q+\bar\epsilon Q)U\Big|_{\substack{\text{no }\theta\\ \text{component}\\ (\theta=0)}}\equiv\boxed{\delta_\epsilon f=\epsilon\phi+\bar\epsilon\bar\chi}$$

- Question: f in the component expansion has no θ's why doesn't it appear here?

- Answer: $f=f(x^m)$ that is $f\neq f(\theta,\bar\theta)$ so $\partial^\alpha f=0$ and $\bar\partial_{\dot\alpha}f=0$. Additionally the $\sigma^m\bar\theta\partial_m$ and $\theta\sigma^m\partial_m$ terms don't vanish when acting on f, but they do have a θ or $\bar\theta$ leftover, hence they don't contribute to $\delta_\epsilon f$. (notice those terms go into $\delta\phi$ and $\delta\bar\chi$)

Chiral & Antichiral Superfields : Φ & $\Phi^\dagger$

We saw the superfield U had $8_B + 8_F = 16$ components this representation of SUSY is in fact reducible,

> **Def^{n}/** Φ is CHIRAL SUPERFIELD if $\bar{D}_{\dot{\alpha}}\Phi = 0$
> $\bar{\Phi}$ is ANTICHIRAL SUPERFIELD if $D_\alpha \bar{\Phi} = 0$

Lets see why $\{Q_\alpha, \bar{D}_{\dot{\beta}}\} = 0$ is important:

$$\begin{aligned} \epsilon^\rho Q_\rho (\bar{D}_{\dot{\alpha}}\Phi) &= \epsilon^\rho(-\bar{D}_{\dot{\alpha}} Q_\rho)\Phi \\ &= \bar{D}_{\dot{\alpha}}(\epsilon^\rho Q_\rho \Phi) \\ &= \bar{D}_{\dot{\alpha}}(\delta\Phi) = 0 \end{aligned}$$

The Chiral S.F. $\Phi \longmapsto \Phi + \delta\Phi$ from the calculation above its clear that $\bar{D}_{\dot{\alpha}}(\Phi + \delta\Phi) = 0$ hence susy takes Chiral S.F. $\longmapsto$ CHIRAL S.F's. The Chiral constraint is called a <u>covariant constraint</u> since it is <u>preserved under susy δ's</u>.

Now you could work out $\bar{D}_{\dot{\alpha}}\Phi = 0$ to find directly what constraints are $\Rightarrow$ for the components:

$$\begin{aligned} &\chi = 0 && \bar{\lambda} = \tfrac{i}{2}(\partial_m \phi)\sigma^m \\ &n = 0 && \psi = 0 \\ &v_m = i\partial_m f && d = -\tfrac{1}{4}\Box f \end{aligned}$$

Its not hard to see $\delta\chi = 0$, $\delta n = 0$, $\delta\psi = 0$ which is just the same covariance realized at the component level. We have reduced the number of components by constraing the S.F. you could count to see $n_B = n_F$ still, but it will be more convenient after we find the general sol^{n} for Φ.

Chiral Superfield's Components

A somewhat sneaky sol^{n} to this problem goes like so, intro. coordinates $y^m = x^m + i\theta\sigma^m\bar{\theta}$ underwhich the supercovariant derivative takes the form

$$D_\alpha = \partial_\alpha + 2i\sigma^m_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial/\partial y^m$$
$$\bar{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}}$$

A nice simplification in view of the following,

$$\boxed{\bar{D}_{\dot{\alpha}} y^m = 0} \quad \text{AND} \quad \boxed{\bar{D}_{\dot{\alpha}}\theta^\beta = 0}$$

$$\bar{D}_{\dot{\alpha}}\Phi(y,\theta) = \frac{\partial\Phi}{\partial y}\bar{D}_{\dot{\alpha}}y + \frac{\partial\Phi}{\partial\theta}\bar{D}_{\dot{\alpha}}\theta = 0$$

Thus a function of y and θ will be a Chiral Superfield!

$$\boxed{\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)}$$

Bosons : A, F : $n_B = 1+1$
Fermions : ψ : $n_F = 2$

- We have constrained Φ so as to make $n_B = n_F$. In fact this is as far as we can go, its not possible to put further restraints on Φ and still have a superfield, hence Φ forms a irreducible representation of Susy.

- Next time we will see $\exists$ another way to constrain a superfield covariantly ... the "Vector Superfield"

- By the way what we have eliminated via $\bar{D}_{\dot{\alpha}}\Phi = 0$ does not on its own form a SUSY rep, the general unconstrained S.F. is <u>not</u> fully reducible. It turns out that the CHIRAL AND VECTOR S.F.'s are the <u>only</u> irreducible rep's of susy for N=1 SUSY in 4-spacetime dimensions.

We obtain $\Phi(x,\theta,\bar\theta)$ by expanding $\Phi(y,\theta)$. Remember $\Phi(y,\theta) = A(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta F(y)$ where $y = x + i\theta\sigma\bar\theta$,

$$\begin{aligned}
A(y) &= A(x + i\theta\sigma\bar\theta) \\
&= A(x) + i\theta\sigma^m\bar\theta\partial_m A(x) + \tfrac{1}{2}(i\theta\sigma\bar\theta)(i\theta\sigma\bar\theta)\partial_m\partial_n A(x) \\
&= A(x) + i\theta\sigma^m\bar\theta\partial_m A(x) - \tfrac{1}{2}\eta^{mn}(\theta\theta)(\bar\theta\bar\theta)\partial_m\partial_n A(x) \\
&= A(x) + i\theta\sigma^m\bar\theta\partial_m A(x) - \tfrac{1}{2}(\theta\theta)\bar\theta\bar\theta\,\Box A(x)
\end{aligned}$$

$$\begin{aligned}
\psi(y) &= \psi(x + i\theta\sigma\bar\theta) \\
&= \psi(x) + i\theta\sigma^m\bar\theta\partial_m\psi(x) + \left(\text{terms that vanish when multiply by } \theta\right)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \sqrt{2}\,\theta\psi(y) &= \sqrt{2}\,\theta\psi(x) + i\sqrt{2}\,\theta(\theta\sigma^m\bar\theta\,\partial_m\psi(x) \\
&= \sqrt{2}\,\theta\psi(x) + i\sqrt{2}\,\theta^\gamma(\theta^\alpha\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m)\psi_\gamma(x) \\
&= \sqrt{2}\,\theta\psi(x) + i\sqrt{2}\left(\tfrac{1}{2}\theta\theta(\partial_m\psi(x)\sigma^m\bar\theta)\right)
\end{aligned}$$

(now its in convential form)

$$\begin{aligned}
F(y) &= F(x + i\theta\sigma^m\bar\theta) \\
&= F(x) + (\text{terms which vanish against } \theta\theta)
\end{aligned}$$

$$\Phi(x,\theta,\bar\theta) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^m\bar\theta\partial_m A(x) + \frac{i}{\sqrt{2}}\theta\theta\partial_m\psi(x)\sigma^m\bar\theta - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\Box A(x)$$

What are these ??? $\longrightarrow$

$$\boxed{\begin{aligned}
\delta A &= \sqrt{2}\,\epsilon\psi \\
\delta\psi &= \sqrt{2}\,\epsilon F + \sqrt{2}\,i\sigma^m\bar\epsilon\partial_m A \\
\delta F &= \sqrt{2}\,i\partial_m\psi\sigma^m\bar\epsilon
\end{aligned}}$$

$(\bar\chi = 0,\ \phi \to \sqrt{2}\psi)$
$(m \to F,\ v_m \to i\partial_m f,\ f \to A$
$(\bar\lambda \to \tfrac{i}{2}\partial_m\phi\sigma^m)$

(Dictionary Between CHIRAL Φ AND U.)

Notational Apology: In lecture 1 we used real scalars A, B and Majorana here we have complex scalars and Weyl Spinors

$$A = \frac{A - iB}{\sqrt{2}}$$

$$\begin{aligned}
\delta A &= \tfrac{1}{\sqrt{2}}(\delta A - i\delta B) \\
&= 1/\sqrt{2}\,(\bar\epsilon\psi + \bar\epsilon\gamma^5\psi) \\
&= 1/\sqrt{2}\,\bar\epsilon(1+\gamma^5)\psi \\
&= 1/\sqrt{2}\,(\epsilon^\alpha\ \bar\epsilon_{\dot\alpha})\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\psi_\alpha\\ \bar\psi^{\dot\alpha}\end{pmatrix} = \sqrt{2}\,\epsilon^\alpha\psi_\alpha
\end{aligned}$$

Likewise we could see the equivalence of this lectures F and F, G before

$$F = \frac{F + iG}{\sqrt{2}}$$

Vector Superfields

The Vector S.F. provides another irreducible representation of SUSY. Like last time we constrain a function on superspace to kill some of the component fields.

Def^n/ V is a vector superfield if V is a scalar superfield that obeys $V = V^\dagger$

Lets see what this means for the component fields,

$$V = f + \theta\phi + \bar\theta\bar\chi + \theta^2 m + \bar\theta^2 n + \theta\sigma\bar\theta v_m + \theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\psi + \theta^2\bar\theta^2 d$$

$$\boxed{V = V^\dagger}$$

$$\left\{\begin{array}{lcl} f &=& f^* \\ \bar\chi &=& \phi^* \\ m &=& n^* \\ v_m &=& v_m^* \\ \bar\lambda &=& \psi^* \\ d &=& d^* \end{array}\right\}$$

this is (76) in Lykken's Review. Probably some $*$'s should read $\dagger$ instead.

Details:

$$\Big((\theta\phi)^\dagger = \bar\theta\bar\phi \Big) \longrightarrow V = V^\dagger \Rightarrow \bar\theta\bar\chi = \bar\theta\bar\phi$$
$$\therefore \bar\chi = \bar\phi$$

Notice that ψ and $\bar\psi$ are related by

$$\bar\psi_{\dot\alpha} \equiv (\psi_\alpha)^\dagger \quad \& \quad \psi^\alpha \equiv (\bar\psi_{\dot\alpha})^*$$

$$\psi^\alpha = (\bar\psi_{\dot\rho})^* \bar\sigma^{0\dot\rho\alpha}$$
$$\bar\psi^{\dot\alpha} = (\psi_\rho)^* \sigma^{0\rho\dot\alpha}$$

(How ψ and $\bar\psi$ are related)

Components of Vectorsuperfield

The expansion below may seem strange, but we will see the utility of this choice of basis in a moment,

C(x) in Wess & Bagger.

$$V(x,\theta,\bar\theta) = f(x) + i\theta\chi - i\bar\theta\bar\chi + \frac{i}{2}\left[m+in\right]\theta\theta - \frac{i}{2}\left[m-in\right]\bar\theta\bar\theta$$

$$-\theta\sigma^m\bar\theta V_m + i\theta\theta\bar\theta\left[\bar\lambda + \frac{i}{2}\bar\sigma^m\partial_m\chi\right] - i\bar\theta\bar\theta\theta\left[\lambda + \frac{i}{2}\sigma^m\partial_m\bar\chi\right]$$

$$+ \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D + \frac{1}{2}\Box f\right]$$

Consider now the sum of a Chiral & AntiChiral Superfield where Φ and $\Phi^\dagger$ are related thru congugation,

$$\Phi + \Phi^\dagger = A + A^* + \sqrt{2}\left(\theta\psi + \bar\theta\bar\psi\right) + \theta\theta F + \bar\theta\bar\theta F^*$$

$$+ i\theta\sigma^m\bar\theta\partial_m(A - A^*) + \frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^m\partial_m\psi$$

$$+ \frac{i}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^m\partial_m\bar\psi + \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\Box\left(A + A^*\right)$$

Easy to see that $\Phi + \Phi^\dagger$ is a vector superfield. Then the transformation $V \longmapsto V + \Phi + \Phi^\dagger$ is sensible, the output is also a vector S.F.

$$f \longmapsto f + A + A^*$$
$$\chi \longmapsto \chi - i\sqrt{2}\psi$$
$$m + in \longmapsto m + in - 2iF$$
$$V_m \longmapsto V_m - i\partial_m(A - A^*)$$ ← like usual transformation
$$\lambda \longmapsto \lambda$$
$$D \longmapsto D$$
"gauge" invariant

$\Phi + \Phi^\dagger$ encodes a gauge transformation in the language of superfields.

$$V \longmapsto V + \Phi + \Phi^\dagger$$ Supergauge Transformation

WESS ZUMINO GAUGE : V_{WZ}

The generalized gauge transformation $V \to V + \Phi + \Phi^\dagger$ seemed to have more than we really needed. This is particularly clear if we examine the vector field in the "WESS ZUMINO GAUGE" $(C = m = n = \chi = 0)$

$$
\begin{aligned}
v_m &\longmapsto v_m - i\partial_m (A - A^*) \\
\lambda &\longmapsto \lambda \\
D &\longmapsto D
\end{aligned}
$$

Recall how SUSY mixed the component fields and it will become clear that SUSY will not preserve the WZ type vector superfield, we need to construct the "SPINOR SUPERFIELD" which has just v_m, λ, D as components. Before that notice a few more things about V_{WZ}

$$V = -\theta\sigma^m\bar{\theta}\, v_m + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\, D$$

VECTOR BOSON OR GAUGE BOSON (v_m)

GAUGINO (SUPERPARTNER) OF GAUGE BOSON ($\bar{\lambda}$, λ)

AUXILLARY FIELD (D)

$V \sim$ Supersymmetric Yang-Mills Potential (or vector potential, here everything ABELIAN)

Just as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (OR $F = dA$) we expect the SPINOR SUPERFIELD OR SUPERFIELD STRENGTH to involve some sort of derivative of V.

SPINOR SUPERFIELDS ; W_α AND $\overline{W}_{\dot\alpha}$

We need a definition which is sensible in view of SUSY transformations, and makes it so W_α has just V_m, λ and D for components.

$$\boxed{\text{Def}^{n}/\quad \begin{aligned} W_\alpha &= -\tfrac{1}{4}(\bar D\bar D)D_\alpha V \quad \text{where } V=V^\dagger \\ \overline{W}_{\dot\alpha} &= -\tfrac{1}{4}(DD)\bar D_{\dot\alpha}V \quad \text{that is } V \text{ is vector S.F.}\end{aligned}}$$

Remark: You can see from acting on the definition,

$$D^\alpha\left(W_\alpha = -\tfrac{1}{4}\bar D\bar D D_\alpha V\right) \qquad \therefore \quad \boxed{\bar D_{\dot\alpha}\overline{W}^{\dot\alpha} = D^\alpha W_\alpha}$$
$$\bar D_{\dot\alpha}\left(\overline{W}^{\dot\alpha} = -\tfrac{1}{4}DD\bar D^{\dot\alpha}V\right)$$

Property: W_α is CHIRAL since $\bar D_{\dot\beta}W_\alpha = -\frac{1}{4}\cancel{\bar D_{\dot\beta}(\bar D\bar D)}^{\,0}D_\alpha V = 0$
Likewise $\overline{W}_{\dot\alpha}$ IS ANTICHIRAL since $D_\beta\overline{W}_{\dot\alpha}=0$

Lets examine how $\overline{W}_{\dot\alpha}$ transforms under a supergauge transformation, meaning $V\longrightarrow V+\Phi+\Phi^\dagger$,

$$\overline{W}_{\dot\alpha} \longmapsto -\tfrac{1}{4}DD\bar D_{\dot\alpha}(V+\Phi+\Phi^\dagger)$$

$$= -\tfrac{1}{4}DD\bar D_{\dot\alpha}V - \tfrac{1}{4}DD\bar D_{\dot\alpha}\Phi - \tfrac{1}{4}DD\bar D_{\dot\alpha}\Phi^\dagger$$

$$= \overline{W}_{\dot\alpha} - \tfrac{1}{4}D^\alpha\left(D_\alpha\bar D_{\dot\alpha} + \bar D_{\dot\alpha}D_\alpha\right)\Phi^\dagger \qquad (\text{added zero} = D_\alpha\Phi^\dagger = 0)$$

$$= \overline{W}_{\dot\alpha} + \tfrac{1}{2}D^\alpha\sigma^m_{\alpha\dot\alpha}P_m\Phi^\dagger \qquad (\text{since } \{D_\alpha,\bar D_{\dot\alpha}\} = -2\sigma^m_{\alpha\dot\alpha}P_m)$$

$$= \overline{W}_{\dot\alpha} + \tfrac{1}{2}\sigma^m_{\alpha\dot\alpha}\left(D^\alpha P_m - P_m D^\alpha\right)\Phi^\dagger \qquad (\text{again } D^\alpha\Phi^\dagger = 0)$$

$$= \overline{W}_{\dot\alpha} + \tfrac{1}{2}\sigma^m_{\alpha\dot\alpha}[D^\alpha, P_m]\Phi^\dagger$$

$$= \overline{W}_{\dot\alpha}$$

$$\boxed{\overline{W}_{\dot\alpha} \text{ AND } W_\alpha \text{ ARE INVARIANT UNDER A SUPERGAUGE TRANSFORMATION}}$$

Components of Spinor Superfield

If you are patient or very bored you might try to work out the componenents of W_α from the definition in the end you would find,

$$W_\alpha = -i\underset{\text{Gaugino}}{\lambda_\alpha} + \theta \underset{\text{Auxillary}}{D} - \tfrac{i}{2}(\sigma^m\bar{\sigma}^n\theta)_\alpha \underbrace{\left[\partial_m V_n - \partial_n V_m\right]} + \theta\theta\,\sigma^m_{\alpha\dot{\beta}}\,\partial_m \bar{\lambda}^{\dot{\beta}}$$

Gaugino   Auxillary

$f_{mn} \equiv \partial_m V_n - \partial_n V_m$
(field strength)

$n_{\text{fermionic}} = 4$ (λ is fermion)
$n_{\text{Bosonic}} = 1 + 3$ (D is scalar and f_{mn} gives 3)
(its clear f_{mn} has 6 <u>real</u> dof $\Rightarrow$ 3 complex dof.)

$$f_{mn} \sim \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & B_2 \\ -E_2 & B_3 & 0 & B_1 \\ -E_3 & B_2 & B_1 & 0 \end{pmatrix} \sim \text{6 indep. components.}$$

<u>Remark</u>: W_α contains just the fields of interest. If you wanted to construct the simplest SUSY model containing a vector boson you would be forced to consider the $\{\lambda, V_m, D\}$ multiplet. In fact superfields can be built from the components using the operator:

$$e^{(\theta Q + \bar{\theta}\bar{Q})}(\text{component field}) \rightarrow \text{more components.}$$

See Wess-Bagger Chapter III, they explicitly build the Wess-Zumino model in this way.

Nonabelian Spinor Superfield

We build W_α so that it contains just the components of the vector superfield of interest (f_{mn}, λ, D).

$$W_\alpha \equiv -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^{V}$$

Examine then how W_α transforms under an non-abelian gauge transformation $e^V \longrightarrow e^{-i\Lambda}e^V e^{i\Lambda^\dagger}$,

$$W_\alpha \longrightarrow W_\alpha' = -\frac{1}{4}\bar{D}\bar{D}\left[e^{-i\Lambda}e^{-V}e^{i\Lambda^+}D_\alpha e^{-i\Lambda^+}e^V e^{i\Lambda}\right]$$

$$= -\frac{1}{4}\bar{D}\bar{D}\, e^{-i\Lambda}e^{-V}D_\alpha\left(e^V e^{i\Lambda}\right)$$

$$= e^{-i\Lambda}\left(-\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^{V}\right)e^{i\Lambda}$$

$$= e^{-i\Lambda}W_\alpha e^{i\Lambda}$$

We can then using the cyclicity of trace see the following lagrangian is invariant under a gauge transformation,

$$\mathcal{L} = \frac{1}{16g^2}\,\mathrm{Trace}\Big(WW\big|_{\theta\theta} + \bar{W}\bar{W}\big|_{\bar{\theta}\bar{\theta}} + \Phi^+ e^V \Phi\big|_{\theta\theta\bar{\theta}\bar{\theta}}$$
$$+\Big[\big(\tfrac{1}{2}m_{ij}\Phi_i\Phi_j + \tfrac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k\big)\big|_{\theta\theta} + h.c.\Big]\Big)$$

must be symmetric tensors with respect to gauge group.

This lagrangian is a model which couples $i=1,2,..,n$ Chiral Superfields to a gauge superfield W_α. This is the building block for the

Minimal Supersymmetric Standard Model

"MSSM"