Berezin Integration

Defined so that the integration is linear and translation invariant. For a single grassman variable this means

$$\int d(\theta+\xi)\, f(\theta+\xi) = \int d\theta\, f(\theta)$$
$$\int d\theta \frac{d}{d\theta} f(\theta) = 0$$

Which leads us to the curious statement that integration is equivalent to differentiation on grassman's,

$$f(\theta) \equiv f_0 + \theta f_1 \quad \Rightarrow \frac{d}{d\theta} f(\theta) = f_1 = \int d\theta\, f(\theta)$$

The formula below is crucial: essentially $\delta(\theta) = \theta$,

$$\boxed{\int d\theta\, \theta = 1}$$

Of course we will need to integrate over several grassman dimensions where $d\theta$ is naturally generalized via the measure of θ^α which is $\epsilon^{\alpha\beta}$

$$d^2\theta = -\frac{1}{4} d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta} \qquad \text{(CHIRAL MEASURE)}$$

$$d^2\bar\theta = -\frac{1}{4} \epsilon^{\dot\alpha\dot\beta} d\bar\theta_{\dot\alpha} d\bar\theta_{\dot\beta} \qquad \text{(ANTICHIRAL MEASURE)}$$

$$d^4\theta = d^2\theta\, d^2\bar\theta = \frac{1}{16} \epsilon_{\alpha\beta} \epsilon^{\dot\alpha\dot\beta} d\theta^\alpha d\theta^\beta d\bar\theta_{\dot\alpha} d\bar\theta_{\dot\beta}$$

$$\int d^2\theta (\theta\theta) = 1$$

$$\int d^2\bar\theta (\bar\theta\bar\theta) = 1$$

integration just picks off the $\theta\theta$, $\bar\theta\bar\theta$ or $\theta\theta\bar\theta\bar\theta$ term.

$$\int d^2\theta\, d^2\underset{(\theta\theta\bar\theta\bar\theta)}{\bar\theta} = \int d^4\theta (\theta\theta\bar\theta\bar\theta) = 1$$

<u>Remark</u>: since integration ~ differentiation notice $\int d^2\theta \sim \frac{\partial^2}{\partial\theta^2}$ which kills every term except the $\theta\theta$ term in a superfield, no θ's remain after this operation!

N=1 Globally Supersymmetric Actions

This is GLOBAL susy because the variation δ acts the same at all points in superspace. To look at LOCAL susy the parameters ϵ must be changed from constant parameters to functions on superspace. Doing that in a physically sensible way is what is called SUPERGRAVITY = SUSY as a local theory.

We will content ourselves with GLOBAL supersymmetry for which we have three constructions we will use again & again...

$$\int d^4x \left[\int d^2\theta \, \Phi(y,\theta) + \int d^2\bar{\theta} \, \Phi^\dagger(y^\dagger,\bar{\theta}) \right] = S_{\text{CHIRAL}}$$

$$\int d^4x \int d^4\theta \, V(x,\theta,\bar{\theta}) = S_{\text{VECTOR}}$$

Φ IS CHIRAL SUPERFIELD
$\Phi^\dagger$ IS ANTICHIRAL SUPERFIELD
V IS VECTOR SUPERFIELD

All of the actions have the property that after the $\theta, \bar{\theta}$ integrations we are left with a integrand which transforms as a total derivative under susy transformation.

$$\boxed{\begin{aligned} \delta S_{\text{CHIRAL}} &= 0 \\ \delta S_{\text{VECTOR}} &= 0 \end{aligned}}$$

WESS ZUMINO MODEL : SUPERFIELD FORMULATION

Consider the action given below. We take Φ to be a CHIRAL superfield and Φ^+ to be ANTICHIRAL,

$$S = \int d^4x \left\{ \int d^4\theta\, \Phi^+\Phi - \int d^2\theta \left(\tfrac{1}{2}m^2\Phi^2 + \tfrac{1}{3}g\,\Phi^3\right) + \text{h.c.} \right\}$$

This action has $\delta S = 0$ since $(\Phi^+\Phi)^+ = \Phi^{++}\Phi^{++} = \Phi^+\Phi$ hence $\Phi^+\Phi$ is a vector superfield. Also Φ^2 and Φ^3 are CHIRAL superfields $\Rightarrow$ $\theta, \bar\theta$ integrations leave just an x-space function which transforms as a total derivative under susy δ $\Rightarrow$ $\delta S = 0$. THIS is the power of the superfield formulation, that paragraph replaces about 5 pages of component field calculations!

$$\boxed{\begin{aligned}\Phi &= A + \sqrt{2}\psi + \theta\theta F + i\theta\sigma^m\bar\theta\partial_m A + \tfrac{i}{\sqrt{2}}(\theta\theta)\partial_m\psi\sigma^m\bar\theta - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\Box A \\ \Phi^+ &= A^* + \sqrt{2}\bar\psi + \bar\theta\bar\theta F^* - i\theta\sigma\bar\theta\partial A^* + \tfrac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^m\partial_m\bar\psi - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\Box A^*\end{aligned}}$$

Let $\sim$ denote the bosonic projection (ignore $\psi, \bar\psi$ for clarity)

$$\tilde\Phi = A + \theta\theta F + i\theta\sigma^m\bar\theta\partial_m A - \tfrac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box A$$
$$\tilde\Phi^+ = A^* + \bar\theta\bar\theta F^* - i\theta\sigma^m\bar\theta\partial_m A^* - \tfrac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box A^*$$

$$\tilde\Phi^+\tilde\Phi\big|_{\theta\theta\bar\theta\bar\theta} = -\tfrac{1}{4}(\Box A^*)A - \tfrac{1}{4}A^*\Box A + F^*F + \tfrac{1}{2}\partial^m A^*\partial_m A$$

$$\int d^2\theta\,\tilde\Phi^2 = 2AF \qquad \left(\int d^2\theta\,\tilde\Phi^2 = \tilde\Phi^2\big|_{\theta\theta}\right)$$
$$\int d^2\theta\,\tilde\Phi^3 = 3A^2F$$

$$\boxed{\tilde S = \int d^4x \left\{ \partial^m A^*\partial_m A + F^*F - (mAF + gA^2F + mA^*F^* + gA^{*2}F^*) \right\}}$$

Now this is precisely the WZ model we had before!

By the way I just partially integrated the $\Phi^+\Phi$ term

$$\partial^m(A^*\partial_m A) = \partial^m A^*\partial_m A + A^*\partial^m\partial_m A$$

So I can trade $\partial^m A^*\partial_m A$ for $-A^*\Box A$ in the action.

Wess Zumino Model with Mass & Coupling Terms

Last time we found that the WZ model which consists of the multiplet $\{A, \psi, F\}$ and SUSY transformations between these, namely:

$$\boxed{\begin{aligned} \delta_\epsilon A &= \sqrt{2}\,\epsilon\psi \\ \delta_\epsilon \psi &= i\sqrt{2}\,\sigma^m \bar{\epsilon}\,\partial_m A + \sqrt{2}\,\epsilon F \\ \delta_\epsilon F &= i\sqrt{2}\,\bar{\epsilon}\bar{\sigma}^m \partial_m \psi \end{aligned}}$$

These transformations leave $S = \int d^4x\, \mathcal{L}$ invariant when the lagrangian has the form,

$$\boxed{\mathcal{L} = \mathcal{L}_0 + m\mathcal{L}_m + g\mathcal{L}_c}$$

$$\mathcal{L}_0 = i\partial_n \bar{\psi}\bar{\sigma}^n \psi + A^* \Box A + F^* F \quad \text{(Kinetic Terms)}$$

$$\mathcal{L}_m = AF + A^* F^* - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi} \quad \text{(Mass Terms)}$$

$$\mathcal{L}_c = A^2 F + A^{*2} F^* + A\bar{\psi}\psi \quad \text{(Coupling Terms)}$$

Notice F is an auxillary field we can eliminate it by imposing it's eq^{n}'s of motion. (notice they are algebraic; Aux. Field)

$$\frac{\partial \mathcal{L}}{\partial F} = F^* - mA - gA^2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial F^*} = F - mA^* - g(A^*)^2 = 0$$

By imposing these eq^{n}'s of motion we can recast the WZ action in terms of the "Superpotential" $V(A, A^*)$

$$V(A, A^*) = |F|^2 = [mA^* + g(A^*)^2][mA + gA^2] \geq 0$$

$$S = \int d^4x \left(i\partial_n \bar{\psi}\bar{\sigma}^n \psi + \partial^m A^* \partial_m A - \underline{V(A^*, A)} - \tfrac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) + A\bar{\psi}\psi \right)$$

Next we will see how all these results fall elegantly from the SUPERFIELD *construction.*

General Chiral Models

We can write a more general action of which WZ is an example.

$$S = \int d^4x \left(\int d^4\theta\, \Phi^\dagger \Phi - \int d^2\theta\, W(\Phi) + \int d^2\bar{\theta}\, W^*(\Phi^\dagger) \right)$$

- $W(\Phi)$ is SUPERPOTENTIAL

Its a HOLOMORPHIC function of Φ

- Again find F is auxillary thus only A, A^* remain after imposing F's eq^{n} of motion

$$V_F(A, A^*) = |F|^2 = \left| \frac{\delta W}{\delta \Phi} \right|^2_{\Phi = A}$$

Lets see how this produces (WZ) model choose

$$W_{WZ}(\Phi) = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3 \quad \text{our choice}$$

$$V_F(A, A^*) = |m\Phi + g\Phi^2|^2_{\Phi = A} = |mA + gA^2|^2$$

Just as Before.

CUBIC $W(\Phi)$ $\Rightarrow$ QUARTIC SCALAR POTENTIAL V $\Rightarrow$ WZ model is most general unitary, renormilizable 4-D SUSY Action for a single CHIRAL S.F.

Complex Scalar Field : Gauge Theory Example

We begin with the Lagrangian and eq^{n}'s of motion:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi \qquad \begin{aligned} (\Box + m^2)\phi &= 0 \\ (\Box + m^2)\phi^* &= 0 \end{aligned}$$

This Lagrangian is invariant under a global $U(1)$ symmetry

$$\phi \longrightarrow e^{-i\Lambda}\phi \quad \text{AND} \quad \phi^* \longrightarrow e^{i\Lambda}\phi^* \qquad (\Lambda \in \mathbb{R})$$

The above simply rotates ϕ at all points in spacetime. Now if this symmetry corresponds to some physical process with dynamics then there is something wrong, the global symmetry should be instead adapted to a local symmetry; these consequences follow,

- Λ replaced with $\Lambda(x)$
- Gauge Potential or Connection A_μ must be introduced
- Covariant Derivatives replace ordinary derivatives
- Pure Gauge term added to $\mathcal{L}$

$$\boxed{\mathcal{L} = (D_\mu \phi)(D^\mu \phi^*) - m^2 \phi^* \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}}$$

$D_\mu \phi^* \equiv (\partial_\mu \bullet ieA_\mu)\phi^* \leftarrow$ Covariant Derivative

$$\begin{pmatrix} \phi \longrightarrow e^{-i\Lambda(x)}\phi \\ \phi^* \longrightarrow e^{i\Lambda(x)}\phi^* \end{pmatrix} \leftarrow \text{local gauge transformations}$$

$$\boxed{A_\mu \longrightarrow A_\mu + \frac{1}{e}\partial_\mu \Lambda}$$

the potential A_μ must transform in this way to maintain the invariance of the action under local gauge transformations.

PURE GAUGE ACTION

Now we will see how to generalize the $F^{\mu\nu}F_{\mu\nu}$ term that occurs in ordinary gauge theory to its supersymmetric analogue. We will let the superfields do the work for us, but first recall,

$$W_\alpha = -i\lambda_\alpha + \theta D - \frac{i}{2}(\sigma^m\bar{\sigma}^n\theta)_\alpha f_{mn} + \theta\theta\sigma^m_{\alpha\dot{\alpha}}\partial_m\bar{\lambda}^{\dot{\alpha}}$$

<u>CLAIM</u>: *the following action is gauge and susy invariant.*

$$S = \frac{1}{2}\int d^4x \underbrace{\int d^2\theta\, W^\alpha W_\alpha}_{W^\alpha W_\alpha|_{\theta\theta}} = \int d^4x \underbrace{\left(-\frac{1}{4}f^{mn}f_{mn} - i\lambda\sigma^m\nabla_m\bar{\lambda} + \frac{1}{2}D^2 + \cdots\right)}_{\substack{\text{follows from multiplying} \\ W^\alpha W_\alpha \text{ out and collecting} \\ \text{just the } \theta\theta \text{ term.} \\ \text{(omitted instanton term)}}}$$

<u>WHY $\delta S = 0$</u>? Now if δ is a supergauge transformation then just recall that W_α was invariant under a supergauge trans. $\therefore$ $W^\alpha W_\alpha$ is unchanged $\therefore$ $\delta S = 0$.

On the other hand if δ is a susy transformation then we need to recall that W_α is a Chiral S.F. hence $W^\alpha W_\alpha$ is a Chiral S.F.. Then consider this

$$\Phi = \tilde{A} + \sqrt{2}\,\theta\tilde{\psi} + \theta\theta\tilde{F} \Rightarrow \Phi|_{\theta\theta} = \tilde{F}$$

$$\delta_\epsilon\tilde{F} = \sqrt{2}\, i\,\partial_m(\psi\sigma^m\bar{\epsilon})$$

THE $\theta\theta$ COMPONENT TRANSFORMS AS A TOTAL DERIVATIVE! So when we integrate over d^4x this vanishes

$$\therefore \boxed{\delta S = 0}$$

$$\delta S = \frac{1}{2}\int_{M} d^4x\, \delta\left(W^\alpha W_\alpha|_{\theta\theta}\right) = \frac{1}{2}\int_{M} d^4x \sqrt{2}\, i\,\partial_m(\cdots) = \sqrt{2}\, i(\cdots)\Big|_{\partial M}$$

But we assume the fields are localized so vanish on ∂M.

Gauge Transformations on Φ

We begin by writing a global symmetry on the CHIRAL superfields Φ_i (we consider several Chiral Superfields)

$$\Phi_i' = \exp(-i t_i \lambda)\Phi_i$$

t_i are $U(1)$ charges
λ are rigid $U(1)$ angles

$$\text{Notice: } \bar{D}_{\dot\alpha}\Phi_i' = \bar{D}_{\dot\alpha}\underbrace{(-i t_i \lambda)}_{\text{constant}}\Phi_i' + \exp(-i t_i\lambda)\underbrace{\bar{D}_{\dot\alpha}\Phi_i}_{\text{zero as } \Phi_i \text{ CHIRAL}} = 0 \quad (\star)$$

So $\bar{D}_{\dot\alpha}\Phi_i' = 0 \Rightarrow$ global $U(1)$ rotation of CHIRAL S.F. produces again a CHIRAL S.F.. That is it respects SUSY. For a collection of Chiral Superfields:

$$\mathcal{L} = \Phi_i^\dagger \Phi_i\Big|_{\theta\theta\bar\theta\bar\theta} + \left[\tfrac{1}{2} m_{ij}\Phi_i\Phi_j + \tfrac{1}{3} g_{ijk}\Phi_i\Phi_j\Phi_k\right]\Big|_{\theta\theta} + \text{h.c.}$$

(with $m_{ij} = 0$ if $t_i + t_j \neq 0$ & $g_{ijk} = 0$ if $t_i + t_j + t_k \neq 0$)
otherwise $\Phi_i'\Phi_j' = \exp(-i(t_i+t_j)\lambda)\Phi_i\Phi_j \neq \Phi_i\Phi_j$
will cause $\mathcal{L}$ to not be invariant.

So we make $\lambda \longrightarrow \Lambda(x)$ This gives us a unique rotation at each point in space, that is a local gauge transformation. From the calculation $(\star)$ above you can see we must take $\Lambda(x)$ to be a CHIRAL superfield in order for the gauge transformation to respect SUSY.

$$\Phi_i' = \exp(-i t_i \Lambda)\Phi_i$$

$$\Phi_i^{\dagger\prime} = \exp(i t_i \Lambda^\dagger)\Phi_i^\dagger$$

These transformations respect SUSY because Φ_i' is a CHIRAL superfield AND $(\Phi_i^\dagger)'$ is an ANTI CHIRAL superfield.

$$(\Phi_i^\dagger)'\Phi_i' = \Phi_i^\dagger\Phi_i\, e^{i t_i(\Lambda^\dagger - \Lambda)} = \Phi_i^\dagger\Phi_i\, e^{t_i V}$$

Gauged Chiral Action:

We determined $\Phi_i^{\dagger\prime}\Phi_i' = \Phi_i^\dagger e^{t_i V}\Phi_i$ with $V = i(\Lambda^\dagger - \Lambda)$. So then we see that in order to keep $\mathcal{L}$ invariant under gauge transformations on the CHIRAL superfields we must,

$$V \longrightarrow V + i(\Lambda - \Lambda^\dagger) \quad (1)$$

$$\Phi_i^\dagger \Phi_i \longrightarrow \Phi_i^\dagger \exp(it_i(\Lambda^\dagger - \Lambda))\Phi_i \quad (2)$$

$$\underline{\Phi_i^\dagger e^{t_i V}\Phi_i} \longrightarrow \Phi_i^\dagger e^{it_i(\Lambda^\dagger-\Lambda)}e^{t_i(V - i(\Lambda^\dagger - \Lambda))}\Phi_i = \underline{\Phi_i^\dagger e^{t_i V}\Phi_i}$$

Gauged $\mathcal{L}$ Invariant

Then we may write Gauged Chiral action which combines the Chiral and Vector Multiplets in a way which makes sense in view of both SUSY & GAUGE SYM.

$$\mathcal{L} = \frac{1}{4}\left(W^\alpha W_\alpha\big|_{\theta\theta} + \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\big|_{\bar\theta\bar\theta}\right) + \Phi_i^\dagger e^{t_i V}\Phi\big|_{\theta\theta\bar\theta\bar\theta}$$
$$+\left\{\left[\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k\right]\Big|_{\theta\theta} + \text{h.c.}\right\}$$

This is invariant under SUSY and the gauge transform. $\Phi' = \exp(it\Lambda)\Phi$, and $V' = V + i(\Lambda - \Lambda^\dagger)$. Additionally it contains no terms of dimension > 4 since $V^3 = 0$. That implies that $\mathcal{L}$ will be renormilizable.

SUPER ELECTRODYNAMICS : A TOY MODEL

We consider 2 chiral superfields Φ_+ and Φ_- these will make up the electron. The lagrangian is:

$$\mathcal{L}_{QED} = \frac{1}{4}\left(WW|_{\theta\theta} + \bar{W}\bar{W}|_{\bar{\theta}\bar{\theta}}\right) + \left(\Phi_+^\dagger e^{eV}\Phi_+ + \Phi_-^\dagger e^{-eV}\Phi_-\right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$
$$+ m\left(\Phi_+\Phi_-|_{\theta\theta} + \Phi_+^\dagger\Phi_-^\dagger|_{\bar{\theta}\bar{\theta}}\right)$$

Where the Superfields Φ_+ and Φ_- transform like,

$$\Phi_+' = e^{-ie\Lambda}\Phi_+ \quad \text{AND} \quad \Phi_-' = e^{ie\Lambda}\Phi_-$$

When we called t; the charge we meant it. The charge of the Φ_+ field is e and the charge of Φ_- is $-e$.

$$\mathcal{L}_{QED} = \Big(\frac{1}{2}D^2 - \frac{1}{4}f_{mn}f^{mn} - i\lambda\sigma^n\partial_n\bar{\lambda} + F_+F_+^* + F_-F_-^* + A_+^*\Box A_+ + A_-^*\Box A_-$$
$$+ i\left(\partial_n\bar{\Psi}_+\bar{\sigma}^n\Psi_+ + \partial_n\bar{\Psi}_-\bar{\sigma}^n\Psi_-\right)\Big) \quad \text{kinetic \& Aux. terms.}$$
$$+ \Big(eV^n\left[\frac{1}{2}\bar{\Psi}_+\bar{\sigma}^n\Psi_+ - \frac{1}{2}\bar{\Psi}_-\bar{\sigma}^n\Psi_- + \frac{i}{2}A_-^*\partial_nA_- - \frac{i}{2}\partial_nA_+^*A_+ - \frac{i}{2}A_-^*\partial_nA_- + \frac{i}{2}A_+^*\partial_nA_+\right]$$
$$- \frac{ie}{\sqrt{2}}\left(A_+\bar{\Psi}_+\bar{\lambda} - A_+^*\Psi_+\lambda - A_-\bar{\Psi}_-\bar{\lambda} + A_-^*\Psi_-\lambda\right) + \frac{e}{2}D\left[A_+^*A_+ - A_-^*A_-\right]$$
$$- \frac{1}{4}e^2V_nV^n\left(A_+^*A_+ + A_-^*A_-\right)$$
$$+ m\left[A_+F_- + A_-F_+ - \Psi_+\Psi_- - \bar{\Psi}_+\bar{\Psi}_- - A_+^*F_-^* + A_-^*F_+^*\right]\Big) \quad \text{mass \& couplings.}$$

$f_{mn} = \partial_{[m}V_{n]}$ is the field strength ~ <u>Photon</u> f_{mn}
λ is the superpartner of photon; the <u>gaugino</u> λ
Ψ_+ and Ψ_- compose a massive Dirac Spinor ; <u>Electron</u> $\Psi_\pm$
A_+ and A_- make the superpartner of electron ; <u>Selectron</u> A
(fields D and F_+, F_- are auxillary fields)

The Standard Model

Color Force   Electroweak Force

Is a gauge theory with gauge group $SU(3)_{COLOR} \otimes \underbrace{SU(2)_{LEFT} \otimes U(1)_{HYPERCHARGE}}$
It contains a number of force carrying particles which we call "vector bosons".

Bosons in S.M.
- gluons: (9) has spin 1 and carries $(8,1,0)$
- W Bosons: (W^+, W^-, W^0) are spin 1 with $(1,3,0)$ representation
- B Bosons: (B^0) also spin 1, carries $(1,1,0)$

The standard model fields carry a representation of $SU(3)_{COLOR} \otimes SU(2)_{LEFT} \otimes U(1)_{HYPERCHARGE}$. The numbers $(8, 1, 0)$ merely indicate that gluons have:

singlet
8-dimensional representation

Its a long story but basically one chooses the standard model field's representation so that they interact in a way consistent with experiment.

Quarks $\rightarrow$ $Q_i = (u\,d), (c\,s), (t\,b)$ (LEFT HANDED SU(2) 2)

Antiquarks $\rightarrow$ $\bar{u}_i = \bar{u}, \bar{c}, \bar{t}$ (RH SU(2) 1)

$\bar{d}_i = \bar{d}, \bar{s}, \bar{b}$ (RH SU(2) 1)

Leptons $\rightarrow$ $L_i = (\nu_e, e), (\nu_\mu, \mu), (\nu_\tau, \tau)$ (LH SU(2) 2)

Antileptons $\rightarrow$ $\bar{e}_i = \bar{e}, \bar{\mu}, \bar{\tau}$

The index i ranges over families (or "generations")
there are no anti nuetrinos... or are there?...

Consider the Kinetic Part of the Standard Model for just the fermionic fields:

$$\mathcal{L} = -i Q^{\dagger i} \bar{\sigma}^{\mu} \partial_\mu Q_i - i \bar{U}^{\dagger i} \bar{\sigma}^{\mu} \partial_\mu \bar{U}_i - i \bar{d}^{\dagger i} \bar{\sigma}^{\mu} \partial_\mu \bar{d}_i$$
$$- i L^{\dagger i} \bar{\sigma}^{\mu} \partial_\mu L_i - i \bar{e}^{\dagger i} \bar{\sigma}^{\mu} \partial_\mu \bar{e}_i + \cdots$$

Notice we have changed notation now "—" means antiparticle. Notice that the standard model contains Chiral Fermions meaning their LH & RH parts transform under seperate representations of gauge groups. To convert to Dirac Spinors one must put in Chiral Projectors in order to write a sensible lagrangian,

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \quad \text{CHIRAL PROJECTOR}$$

$$\psi_{DIRAC} = \begin{pmatrix} \chi^\alpha \\ \bar{\xi}_{\dot{\alpha}} \end{pmatrix} \quad \text{DIRAC CONTAINS LH \& RH PARTS !!!}$$

$$P_L \psi_{DIRAC} = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix} \quad \chi_\alpha \text{ LH Weyl Spinor}$$

$$P_R \psi_{DIRAC} = \begin{pmatrix} 0 \\ \bar{\xi}_{\dot{\alpha}} \end{pmatrix} \quad \bar{\xi}_{\dot{\alpha}} \text{ RH Weyl Spinor}$$

$$\mathcal{L}_{DIRAC} = -i \bar{\psi}_D \not{\partial} \psi_D - m \bar{\psi}_D \psi_D = -i \bar{\xi} \bar{\sigma}_\mu \partial^\mu \xi - i \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi - m(\xi\chi + \bar{\xi}\bar{\chi})$$

In order to keep LH things interacting with LH things we must kill the RH part otherwise we have unwanted term,

$$\Phi_1 = \begin{pmatrix} \xi_1 \\ \bar{\chi}_1 \end{pmatrix} \qquad \bar{\Phi}_1 P_L \Phi_2 = \chi_1 \xi_2 \quad \text{(Axial Couplings)}$$

$$\bar{\Phi}_1 P_R \Phi_2 = \bar{\xi}_1 \bar{\chi}_2 \qquad \Phi_1 \Phi_2$$

$$\Phi_2 = \begin{pmatrix} \xi_2 \\ \bar{\chi}_2 \end{pmatrix} \qquad \bar{\psi}_1 \gamma^\mu P_L \psi_2 = \bar{\psi}_{1\dot{\alpha}} \sigma^{\dot{\alpha}\beta\mu} \psi_{2\beta}$$

SUPERFIELD MULTIPLETS: VECTOR VS. CHIRAL

The two superfields we have investigated have very different field content. Lets expand on that,

CHIRAL MULTIPLET	Φ	2 real or 1 complex scalar 1 Weyl Fermion ($m \geq 0$)	$\begin{pmatrix} A \\ \psi \end{pmatrix}$
VECTOR MULTIPLET	W	1 massless vector boson 1 Weyl Fermion ($m = 0$)	$\begin{pmatrix} V_m \\ \psi \end{pmatrix}$

VECTOR MULTIPLET

1. Gauge Boson transforms in adjoint rep. ⇒ that the gaugino's are also in adjoint rep. So in this multiplet members share a common gauge group rep.

2. Adjoint rep. is its own conjugate thus fermions in the vector multiplet have the same gauge transformation properties for LH & RH components.

CHIRAL MULTIPLET

1. Unlike Vector Multiplet we have possibility that LH and RH parts transforms under different gauge group representations.

   ⇒ Standard Model Fermions should be put into Chiral multiplets.

Additionally S.M. fermions cannot go into a vector multiplet due to LH/RH asymmetry. Finally we must put the S.M. bosons into a vector multiplet. So these are the guidelines for building the MSSM. the simplest extension of the S.M. incorporating SUSY.

Gauge Structure of S.M. particles

- left handed & RH pieces of quarks and leptons are seperate 2-component Weyl Fermions, these get <u>different</u> gauge tranformation properties.
  ∴ <u>each gets its own complex scalar superpartner</u>

- The squarks and sleptons share same symbol as their superpartner except add ~ to identify

  Note: $\tilde{e}_L$ and $\tilde{e}_R$ are spin 0 particles the L and R refer to the helicity of the superpartners e_L and e_R.

- The s-particles share the same S.M. gauge interactions as their superpartners.

  $\tilde{U}_L$ couples to W boson
  $\tilde{U}_R$ will not.

16) Chiral Supermultiplets in MSSM

Names	Chiral multiplet's names	Spin 0	Spin 1/2	Representation of $SU(3)_C, SU(2)_L, U(1)_Y$
Squarks, quarks (3 families)	Q	$(\tilde{u}_L\ \tilde{d}_L)$	$(u_L\ d_L)$	$(3, 2, \frac{1}{6})$
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3}, 1, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3}, 1, \frac{1}{3})$
sleptons, leptons (x3 families)	L	$(\tilde{\nu}\ \tilde{e}_L)$	$(\nu\ e_L)$	$(1, 2, -\frac{1}{2})$
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(1, 1, 1)$
Higgs, Higgsinos	H_u	$(H_u^+\ H_u^0)$	$(\tilde{H}_u^+\ \tilde{H}_u^0)$	$(1, 2, \frac{1}{2})$
	H_d	$(H_d^0\ H_d^-)$	$(\tilde{H}_d^0\ \tilde{H}_d^-)$	$(1, 2, -\frac{1}{2})$

- The left handed Chiral Fermions transform as doublets under $SU(2)_L$ while the right handed Chiral fermions are all $SU(2)_L$ singlets.
- Y is the hypercharge
- Nuetrinos are all left handed so we drop the L subscript.
- e_L is the electron it has spin 1/2 as we know
  $\tilde{e}_L$ is the selectron it has spin 0, maybe we find
- All of the objects above can be assembled either from a Chiral Superfield or a pair of Chiral Superfields which carry the appropriate gauge rep.
- The "S" stands for scalar, all ~ objects are unobserved presently.
- The inclusion of the higgs as prescribed above is a subtle and delicate issue, we discuss on next pages (3 pages later)

Vector Supermultiplets in MSSM

Names	spin ½	Spin 1	$SU(3)_c$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\tilde{g}$	g	(8, 1, 0)
winos, W bosons	$\tilde{W}^{\pm}$ $\tilde{W}^{0}$	$W^{\pm}$ W^{0}	(1, 3, 0)
bino, B bosons	$\tilde{B}^{0}$	B^{0}	(1, 1, 0)

- Vector Bosons (spin 1 gauge bosons) of S.M clearly must reside in the Vector Multiplets.
- Fermionic Superpartner adds an "ino" to name

g = gluon $\tilde{g}$ = gluino for $SU(3)_{color}$
8 of these guys.

W^+, W^0, W^-, B^0 are gauge bosons of Electroweak symmetry $SU(2)_L \otimes U(1)_Y$

Superpartners $\tilde{W}^{\pm}, \tilde{W}^{0}$ = Winos
$\tilde{B}^{0}$ = Bino

- Electroweak symmetry breaking W^0 and B^0 gauge eigenstates mix to give mass eigenstates Z^0 ($m \neq 0$) γ ($m = 0$) the corresponding gaugino mixtures are called

$\tilde{Z}^0$ = the zino $\tilde{\gamma}$ = photino

THE HIERARCHY PROBLEM : WHY is m_H so small? 44

Standard Model Higgs Potential

$$V = m_H^2 |H|^2 + \lambda |H|^4$$

Requires a non-vanishing vacuum expectation value

$$\langle H \rangle = \sqrt{-m_H^2/2\lambda} \quad \text{and} \quad \langle H \rangle = 174\,\text{GeV from experiment}$$

$$\Rightarrow \quad m_H \approx 100\ \text{GeV}$$

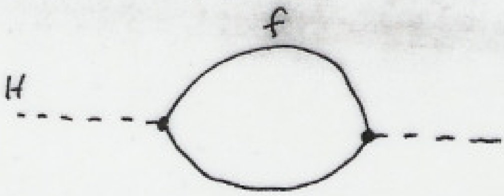
- But m_H recieves enormous quantum corrections from every particle that couples to it
- For fermion with mass m_f

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_f^2 \ln(\Lambda_{UV}/m_f) + \cdots\right]$$

ultraviolet momentum cutoff to regulate loop integral

other terms dep. on way cutoff performed (finite terms) compared to Λ_{UV}

f = many S.M. fermions



- For heavy scalar particle, coupling $-\lambda_S |H|^2 |S|^2$ in Lagrangian,

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \ln(\Lambda_{UV}/m_S) + \cdots\right]$$