Same instructions as Mission 1. Thanks!

Problem 201 Find the flux of $\vec{F}(x,y,z) = \langle xy^2, y^3, zy^2\rangle$ through the surface of the positively oriented cube $E = [0,1]^3$. That is calculate $\int_{\partial E} \vec{F} \cdot d\vec{S}$.

Problem 202 Let $\vec{F} = \langle x^3, y^3, z^3\rangle$. Let S_R be the sphere of radius R. Calculate $\iint_{S_R} \vec{F} \bullet d\vec{S}$.

Problem 203 Let $\vec{F}(x,y,z) = \langle x^3, y^2, z\rangle$ and calculate the flux through the outward-oriented closed surface S bounded by $x^2+y^2 = 1$ and $z = 1$ by $z = x^2+y^2$.

Problem 204 Let E be the cube $[-1,1]^3$. Calculate the flux through ∂E of the vector field

$$\vec{F}(x,y,z) = \big\langle y-x, z-y, y-x\big\rangle$$

(please use the divergence theorem!)

Problem 205 Let E be the set of (x,y,z) such that $x^2+y^2 \leq 4$ and $0 \leq z \leq x^2+y^2$. Find the flux through ∂E of the vector field $\vec{F}$ given below:

$$\vec{F}(x,y,z) = \langle y, xy, -z\rangle$$

(please use the divergence theorem)

Problem 206 Suppose E is the spherical shell $R_1 \leq \rho \leq R_2$ and suppose $\vec{F}(x,y,z) = \nabla \times \vec{A}$ for some everywhere smooth vector field $\vec{A}$. Show that the flux through $\rho = R_1$ is the same as the flux through $\rho = R_2$ by applying the divergence theorem to the spherical shell.

Problem 207 Let S be the pseudo-tetrahedra with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$. Three of the faces of S_1 are subsets of the coordinate planes call these S_{xy}, S_{zx}, S_{yz} with the obvious meanings and call S_T the top face. Let $\vec{F} = \langle y, -x, y\rangle$ and define $\vec{G} = \nabla \times \vec{F}$.

(a.) Calculate the circulation of $\vec{F}$ around each face .

(b.) Calculate the flux of $\vec{G}$ through each face.

(c.) do you see any pattern?

Problem 208 Let $\vec{F}(x,y,z) = \langle x^2, 2x, z^2\rangle$. Calculate $\int_E \vec{F} \bullet d\vec{r}$ where E is the CCW oriented ellipse $4x^2+y^2 = 4$ with $z = 0$. (use Stokes' Theorem)

Problem 209 Let $\vec{F}(x,y,z) = \langle y^2+z^2, x^2+z^2, x^2+y^2\rangle$. Find the work done by $\vec{F}$ around the CCW (as viewed from above) triangle formed from the intersection of the plane $x+y+z = 1$ and the coordinate planes. (use Stokes' Theorem)

Problem 210 Let $\vec{F}(x,y,z) = \langle y^2+z^2, x^2+y^2, x^2+y^2\rangle$. Find the work done by $\vec{F}$ around the CCW-oriented square bounded by $x = \pm 1$ and $y = \pm 1$ in the $z = 0$ plane (use Stokes' Theorem).

Problem 211 Let $\vec{F} = \langle 2x, 2y, 2z\rangle$ and suppose S is a simply connected surface with boundary ∂S a simple closed curve. Show by Stokes' theorem that $\int_{\partial S} \vec{F} \bullet d\vec{r} = 0$

Problem 212 Suppose S is the union of the cylinder $x^2+y^2=1$ for $0 \leq z \leq 1$ and the disk $x^2+y^2 \leq 1$ at $z=1$. Suppose $\vec{F}$ is a vector field such that

$$\nabla \times \vec{F} = \left\langle \sinh(z)(x^2+y^2), ze^{xy+\cos(x+y)}, (xz+y)\tan^{-1}(z) \right\rangle.$$

Calculate the flux of $\nabla \times \vec{F}$ though S. Or, explain why this formula is impossible.

Problem 213 Let S be part of the cylinder $x^2+z^2=1$ where $x \geq 0$ and $0 \leq y \leq 1$. Parametrize S such that its normal field points in the positive x-direction. Calculate $\iint_S \vec{F} \bullet d\vec{S}$ for $\vec{F} = \langle 1, e^{xy}, 3 \rangle$.

Problem 214 Let $\vec{F}(x,y,z) = \langle -x^2y, xy^2, \cos(xz^3) \rangle$.

(a.) Calculate $\nabla \times \vec{F}$.

(b.) Calculate the flux of $\nabla \times \vec{F}$ through the upward oriented open cone $z=r$ for $0 \leq z \leq 2$.

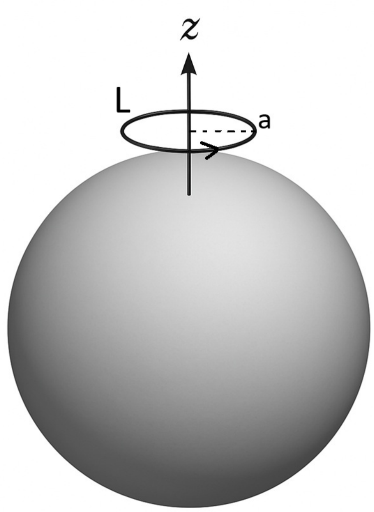
Problem 215 Let $\vec{F}(x,y,z) = \left\langle \dfrac{x-a}{d^3}, \dfrac{y-b}{d^3}, \dfrac{z-c}{d^3} \right\rangle$ where $d = \sqrt{(x-a)^2+(y-b)^2+(z-c)^2}$

(a.) Calculate, and simplify, $\nabla \bullet \vec{F}$.

(b.) Let $S_R(a,b,c)$ be the outward-oriented sphere of radius R centered at (a,b,c). Calculate the flux of $\vec{F}$ through $S_R(a,b,c)$.

(c.) Let S be any, possibly lumpy, closed surface which encloses (a,b,c) in its interior. Calculate the flux of $\vec{F}$ through S.

Problem 216 Consider the vector field $\vec{A}(x,y,z) = \frac{1}{\rho(z-\rho)}\langle -y, x, 0 \rangle$. After considerable calculation it can be shown that $\nabla \times \vec{A} = \frac{\widehat{\rho}}{\rho^2}$ on the domain of $\vec{A}$ which is all of space except the non-negative z-axis. Calculate $\int_L \vec{A} \bullet d\vec{r}$ for the loop L pictured where we assume $a << R$ (unlike the picture, we are thinking of a tiny loop about the positive z-axis)



Problem 217 Suppose $\vec{F}(x,y,z) = \langle -y, x, z^3 \rangle$ and let H be the open hemisphere $x^2+y^2+z^2 = 1$ for $z \geq 0$ oriented in the direction of increasing ρ. Verify Stokes' Theorem by calculating both of the following directly:

(a.) Calculate $\int_{\partial H} \vec{F} \bullet d\vec{r}$,

(b.) Calculate $\int_H (\nabla \times \vec{F}) \bullet d\vec{S}$.

Problem 218 Let D be the disk $x^2 + y^2 \leq 1$ in the $z = 0$ plane oriented in the positive z-direction. Let H be the hemisphere $x^2 + y^2 + z^2 = 1$ oriented outward from the origin. Suppose $\vec{F}$ is a smooth vector field with $\nabla \bullet \vec{F} = 30/\pi$ and $\int_D \vec{F} \bullet d\vec{S} = 22$. Find flux of $\vec{F}$ through H.

Problem 219 Suppose $\vec{F} = \rho^n \widehat{\rho}$ and you are given that the flux of $\vec{F}$ through the sphere of radius R is 4π for every $R > 0$. Show that $n = -2$.

Problem 220 Suppose f is a nonzero function such that ∇f is perpendicular to $\vec{G}$ whereas $\nabla \bullet \vec{G} = \frac{1}{f}\sqrt{x^2 + y^2 + z^2}$. Calculate the flux of $\vec{H} = f\vec{G}$ through the sphere of radius R.