Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. Full credit for full solutions. The grader should be able to understand the solution without looking at the problem statement. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 1 Let $\vec{A} = \langle 0, 1, 3\rangle$ and $\vec{B} = \langle -1, 6, 7\rangle$. Calculate:

(a.) $\vec{A} \bullet \vec{B}$

(b.) the angle between $\vec{A}$ and $\vec{B}$ in degrees

(c.) $\vec{A} \times \vec{B}$

Problem 2 Find the standard angle and magnitude of each vector:

(a.) $\vec{A} = \langle\ 1, \sqrt{3}\ \rangle$,

(b.) $\vec{B} = -\,\widehat{x} - \,\widehat{y}$.

Problem 3 Let $\vec{V}$ be a vector which makes an angle of 120^o with the positive x-axis, 60^o with the positive y-axis and 45^o with the positive z-axis. In addition, you are given $\vec{V} \bullet \widehat{x} = -10$. Find $\vec{V}$.

Problem 4 The angle between $\vec{A}$ and the positive x-axis is 45^o and the angle between $\vec{A}$ and the positive y-axis is 60^o. If the magnitude of $\vec{A}$ is given by $A = 2$ then find the possible form(s) of $\vec{A}$.

Problem 5 Let $\vec{A}, \vec{B}, \vec{C}$ be three dimensional vectors and let λ be a scalar. Simplify the expressions below as much as possible, or explain why the expression is not-defined

(a.) $(\vec{A} \bullet \vec{B}) \times \vec{C}$,

(b.) $\vec{A} \times (\lambda\vec{A})$,

(c.) $(\vec{A} + \lambda\vec{B}) \bullet (\vec{A} \times \vec{B})$,

(d.) $\vec{A} + \vec{A} \bullet \vec{B}$,

(e.) $\vec{A} + (\vec{A} \bullet \widehat{x})\,\widehat{x} + (\vec{A} \bullet \widehat{y})\,\widehat{y} + (\vec{A} \bullet \widehat{z})\,\widehat{z}$,

(f.) $\vec{A} \bullet (\vec{B} + \vec{C}) - \vec{B} \bullet \vec{A}$,

(g.) $(\vec{A} + \vec{B}) \times (3\vec{B} - \vec{A})$,

(h.) $\vec{A} \times \vec{B} + \vec{A} \bullet \vec{B}$.

Problem 6 Let $\vec{A} = \langle 4, 6, 8\rangle$ and $\vec{B} = \langle -1, 2, 2\rangle$.

(a.) Find vectors $\vec{v}_1$ parallel to $\vec{B}$ and $\vec{v}_2$ perpendicular to $\vec{B}$ such that $\vec{A} = \vec{v}_1 + \vec{v}_2$.

(b.) Consider the line through the origin with direction vector $\vec{B}$. How far is the point $(4, 6, 8)$ from the line ?

Problem 7 Let $\vec{A} = \langle 2, 2, 1\rangle$ and $\vec{B} = \langle 1, 1, -4\rangle$. Find $\vec{C}$ with length 1 which is orthogonal to both $\vec{A}$ and $\vec{B}$. Then, find c_1, c_2, c_3 for which $\langle 3, 4, 5\rangle = c_1\vec{A} + c_2\vec{B} + c_3\vec{C}$.

Problem 8 Suppose that $\vec{A}$ and $\vec{B}$ are orthogonal vectors with $A = B$. Show that $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are also orthogonal.

Problem 9 Let λ be a scalar and $\vec{A}, \vec{B}, \vec{C}$ vectors in $\mathbb{R}^n$. Show the following identities are true:

(a.) $\vec{A} \bullet (\lambda\vec{B} + \vec{C}) = \lambda\vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$. ($n$ is an arbitary positive integer)

(b.) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$. ($n = 3$ since the standard cross product requires it)

Please consider using the index notation given in my notes. For example,

$$\vec{A} \bullet \vec{B} = \sum_{i=1}^{n} A_i B_i$$

for n-vectors whereas

$$\vec{A} \times \vec{B} = \sum_{i,j,k=1}^{3} \epsilon_{ijk} A_i B_j \hat{x}_k.$$

Problem 10 For each solution set or parametrization, determine if the object described is a line or a plane. If it is a line then find its direction vector. If it is a plane then find its normal vector. Your answers should use words and equations.

(a.) $\vec{r}(t) = (8 + t, 7, 6 - 13t)$

(b.) $\vec{r}(u, v) = (3 + u + v, 2 - u - v, 100 + u)$

(c.) $3x - 17 + y = 13z$

(d.) $3x - 1 = 2y + 4 = 13z - 8$

(e.) $\vec{r}(s) = (6, 7) + t\langle 3, 4\rangle$

Problem 11 Let $\mathcal{P}_1$ be the plane through the origin which contains $(1, 2, 3)$ and $(3, 3, -2)$. Let $\mathcal{P}_2$ be the plane given by $3x + y - z = 10$. Find the parametrization of the line which is based at $(6, 7, 0)$ and is parallel to the line of intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$.

Problem 12 Consider the points P, Q, R given below. Determine if they form the vertices of a triangle. If not, explain how you know they do not. If so, find the area of the triangle as well as its interior angles.

(a.) $P = (1, 1, 1)$ and $Q = (0, 3, 7)$ and $R = (2, 5, 9)$

(b.) $P = (1, 1, 1)$ and $Q = (1, 3, 3)$ and $R = (1, -2, -2)$

Problem 13 Consider the plane $\mathcal{P}_1$ parametrized by $\vec{r}(s, t) = (1, 2, 3) + s\langle 0, 1, -1\rangle + t\langle 1, 2, 2\rangle$.

(a.) The plane $\mathcal{P}_2$ is given by $4x - y - z = 0$. What is the distance from $\mathcal{P}_1$ to $\mathcal{P}_2$?

(b.) The plane $\mathcal{P}_3$ is given by $3x + 2y + z = 10$. What is the distance from $\mathcal{P}_1$ to $\mathcal{P}_3$?

(c.) What is the distance between $\mathcal{P}_2$ and $\mathcal{P}_3$?

Problem 14 Find the point on the plane $3x - 5y + 2z = 10$ which is closest to:

(a.) the point $(-2, 12, 0)$,

(b.) the line with points $(1, 2, 3)$ and $(4, 4, 4)$.

Problem 15 Let line ℓ_1 have point P_1 and direction vector $\vec{v}_1$ and let the line ℓ_2 have point P_2 and direction vector $\vec{v}_2$. Let $\overrightarrow{P_1P_2}$ denote the vector from P_1 to P_2. We say ℓ_1 and ℓ_2 are **skew** if $\vec{v}_1$ and $\vec{v}_2$ are not colinear. Suppose the lines are skew and show the distance between them is given by:

$$d = \frac{\left|\overrightarrow{P_1P_2} \bullet (\vec{v}_1 \times \vec{v}_2)\right|}{\|\vec{v}_1 \times \vec{v}_2\|}.$$

Problem 16 Let ℓ_1 be the line which contains the points $(1, 1, 1)$ and $(4, 0, 5)$. Let ℓ_2 be the line given by $\vec{r}(t) = (3, 2, 1) + t\langle 4, 0, 7\rangle$ for all $t \in \mathbb{R}$. What is the distance between these lines ?

Problem 17 The purpose of this problem is practice with the cross product with the aim of understanding something about the interplay between two and three dimensional vectors.

(a.) Let $\vec{u} = \langle a, b\rangle$. Calculate $\langle a, b, 0\rangle \times \langle 0, 0, 1\rangle$. How does your result relate to $\vec{u}$?

(b.) Let $\vec{v} = \langle a, c\rangle$. Calculate $\langle a, 0, c\rangle \times \langle 0, 1, 0\rangle$. How does your result relate to $\vec{v}$?

(c.) Let $\vec{w} = \langle b, c\rangle$. Calculate $\langle 0, b, c\rangle \times \langle 1, 0, 0\rangle$. How does your result relate to $\vec{w}$?

Remark: *there is a much simpler method to achieve the result of the calculations above. Given a two-dimensional vector $\langle A, B\rangle$, how can you create a perpendicular vector to the given vector ?*

Problem 18 Formulas for determinants are given in my Lecture notes.

(a.) Let $\vec{v} = det\begin{pmatrix} \widehat{x} & \widehat{y} \\ a & b \end{pmatrix}$ and show $\vec{v} \perp \langle a, b\rangle$.

(b.) Let $\vec{w} = det\begin{pmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 & \hat{x}_4 \\ 2 & 2 & 1 & 0 \\ -2 & 1 & 2 & 0 \\ 1 & -2 & 2 & 0 \end{pmatrix}$ and show $\vec{w}$ is perpendicular to $\vec{a} = \langle 2, 2, 1, 0\rangle$ and $\vec{b} = \langle -2, 1, 2, 0\rangle$ and $\vec{c} = \langle 1, -2, 2, 0\rangle$. How is the magnitude $w = \sqrt{\vec{w} \bullet \vec{w}}$ related to the magnitudes of $\vec{a}, \vec{b}, \vec{c}$?

Remark: *the result seen in (b.) occurred because $\vec{a}, \vec{b}, \vec{c}$ were perpendicular.*

Problem 19 Suppose that A, B, C, D are points in $\mathbb{R}^3$ where no three of these points lie on the same line. Furthermore, suppose S is the quadrilateral $ABCD$ formed from these points. Let M_1, M_2, M_3, M_4 be the midpoints of of the edges of S. Use a vector argument to show $M_1M_2M_3M_4$ is a parallelogram!

Problem 20 Show that a triangle inscribed in a semicircle must be a right triangle. To say the triangle is inscribed in a semicircle means one side of the triangle lies on the diameter of the circle.