Same instructions as Mission 1. Thanks!

Problem 21 Find the Cartesian equation of the surface parametrized by

$$\vec{r}(\alpha, \beta) = \langle \cos\beta \cosh\alpha, \cos\beta \sinh\alpha, \sin\beta \rangle.$$

What did we call such a surface ?

Problem 22 Consider $S = \{(x, y, z) \in \mathbb{R}^3 \mid x + 2y + 3z = 6\}$

(a.) Identify this surface, what is it ? Make a rough sketch.

(b.) Find a parametrization of S which uses x, y as parameters.

(c.) Let $S_1 = \{(x, y, z) \in S \mid x^2 + y^2 \leq 1\}$. Find a parametrization of S_1 which has domain for the parameters of $[0, 2\pi] \times [0, 1]$.

Problem 23 Consider $S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + 3y^2 = 1\}$

(a.) Identify this surface, what is it ? Make a rough sketch.

(b.) Find a parametrization of S which does not involve square roots.

(c.) Let $S_R = \{(x, y, z) \in S \mid y \geq 0\}$. Find a parametrization of S_R which is formed by approrpriately limiting the domain of the parameters in part (b.)

Problem 24 Consider $S = \{(x, y, z) \in \mathbb{R}^3 \mid (x-1)^2 + y^2 + (z+3)^2 = 4\}$

(a.) Identify this surface, what is it ? Make a rough sketch.

(b.) Find a parametrization of S which does not involve square roots.

(c.) Let $S_F = \{(x, y, z) \in S \mid x \geq 1\}$. Find a parametrization of S_F which is formed by approrpriately limiting the domain of the parameters in part (b.)

Problem 25 For each surface below, identify the surface, make a rough sketch and finally provide a parametrization which does not use square roots. Please indicate the domain of your parameters as well and indicate if your parametrization does not cover the whole surface.

(a.) $x^2 + y^2 - z^2 = 0$

(b.) $x^2 - 2y^2 - 9z^2 = 1$

(c.) $x^2 + y^2 - 4z^2 = 1$

Problem 26 Suppose C is a circle of radius R which rests on the plane with equation $x + 2y + 4z = 13$. The center of the circle is at $(3, 1, 2)$. Suppose the circle goes counterclockwise with respect to the axis $\langle 1, 2, 4 \rangle$. Find a parametrization of C where $t = 0$ puts your path at a point in the $\langle 0, 2, -1 \rangle$-direction relative to the center of the circle.

Problem 27 The equations and inequalities below make use of standard cylindrical and spherical coordinates as given in the notes. Determine the curve or surface described and give a rough sketch.

(a.) $r^2 + z^2 = 4$

(b.) $\rho = 1$ with $0 \leq \phi \leq \pi/4$

(c.) $\theta = \pi/3$ and $\rho = 2$

(d.) $r = 3$ and $z = 2$ with $0 \leq \theta \leq \pi/2$

Problem 28 Parametrize the curve of intersection as instructed:

(a.) $x + y + z = 0$ and $\rho = 2$ using θ as a parameter,

(b.) $z = 3$ and $\phi = \pi/4$ using θ as a parameter,

(c.) $\rho = 3$ and $\theta = \pi/6$ using ϕ as a parameter,

Problem 29 Consider the vector $\vec{A} = \widehat{x} + 3\widehat{y} - \widehat{z}$.

(a.) express $\vec{A}$ in terms of the cylindrical frame $\widehat{r}$, $\widehat{\theta}$, $\widehat{z}$

(b.) express $\vec{A}$ in terms of the spherical frame $\widehat{\rho}$, $\widehat{\phi}$, $\widehat{\theta}$.

Remark: see my notes for discussion of the cylindrical and spherical frames. Unlike the Cartesian frame $\widehat{x}$, $\widehat{y}$, $\widehat{z}$, the unit-vectors $\widehat{r}, \widehat{\rho}, \widehat{\theta}, \widehat{\phi}$ have a dependence on the basepoint of the vector.

Problem 30 Let $\vec{r}(t) = \langle \sin t, e^{-t}, t^2 \rangle$. Find:

(a.) $\dfrac{d\vec{r}}{dt}$

(b.) the parametrization of the tangent line to the given path at $(0, 1, 0)$

Problem 31 Let $\vec{A} = \left\langle \dfrac{1}{t}, \sqrt{t}, \ln(t) \right\rangle$ and suppose $\vec{B}$ is a constant vector. Calculate (leave answers in terms of $\vec{B}$ and $\bullet$ and $\times$ as appropriate)
:

(a.) $\frac{d}{dt}\vec{A} \bullet \vec{B}$,

(b.) $\dfrac{d}{dt}\vec{A} \times \vec{B}$,

(c.) $\displaystyle\int_0^1 \left(\vec{A} + \frac{1}{1+t^2}\vec{B} \right) dt$.

Problem 32 For each path below, find the parametrization of the colliding tangent line at $t = t_0$,

(a.) $\vec{\gamma}(t) = \langle t, t^2, t^3 \rangle$ with $t_0 = 1$

(b.) $\vec{\gamma}(t) = \langle t\cos t, t\sin t, t^2 \rangle$ with $t_0 = \pi/2$

(c.) $\vec{r}(t) = \langle t^2 \ln(t+1), \sqrt{t+3} \rangle$ with $t_0 = 0$

Problem 33 Calculate the following given that $\vec{A}(t) = \langle 1, t, t^2 \rangle$ and $\vec{B}(t) = \langle e^{\sin t}, 2^t, \ln(t+2) \rangle$

(a.) $\frac{d}{dt}\left[\cos(t^2)\vec{A} \right]$

(b.) $\frac{d}{dt}\left[\langle 1,2,3\rangle \bullet \vec{A}\right]$

(c.) $\frac{d}{dt}\left[\vec{B}\times\langle 1,2,3\rangle\right]$ (you may leave answer as cross product)

(d.) $\frac{d}{dt}(\vec{A}\circ g)(t)$ where $g(t)=\sin t$

(e.) $\frac{d^2\vec{A}}{dt^2}$ and $\frac{d^2\vec{B}}{dt^2}$

Problem 34 Let $\vec{B},\vec{C}$ be a constant vectors. You should use $\vec{B},\vec{C}$ in your answers as appropriate. Calculate:

(a.) $\frac{d}{dt}\left[t\vec{B}+t^2\vec{C}\right]$

(b.) $\frac{d}{dt}\left[\langle t,t^2,t^3\rangle\times\vec{C}\right]$

(c.) $\vec{B}\bullet\int_0^1\left(t\sin(t^2)\vec{C}\right)\,dt$

(d.) $\int\left(\vec{B}+e^{-t}\vec{C}\right)\,dt$

Problem 35 Find the colliding tangent line to $\vec{\gamma}(t)=\langle t^3-1,t^2,t^4\rangle$ at the point $(0,1,1)$. At what angle(s) does this line intersect the sphere $\rho=3$?

Problem 36 Prove $\frac{d}{dt}\int_a^t\vec{F}(u)\,du=\vec{F}(t)$ where $\vec{F}=\langle F_1,F_2,F_3\rangle$ has continuous component functions.

Problem 37 Suppose $\vec{A}$ and $\vec{B}$ are both differentiable 3-vector-valued functions of time t. Prove the following identities:

(a.) $\frac{d}{dt}(\vec{A}\bullet\vec{B})=\frac{d\vec{A}}{dt}\bullet\vec{B}+\vec{A}\bullet\frac{d\vec{B}}{dt}$

(b.) $\frac{d}{dt}(\vec{A}\times\vec{B})=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$

Problem 38 Suppose $\|\vec{A}(t)\|=1$ for all t. Show $\vec{A}$ and $d\vec{A}/dt$ are orthogonal.

Problem 39 Find the arclength function based at $t=0$ in terms of an integral for each paths below. Then, if possible with techniques from Calculus II, solve the integral:

(a.) $\vec{r}(t)=\langle t\cos e^t,t\sin e^t\rangle$

(b.) $\vec{r}(t)=\langle t,t^2,t^3\rangle$

(c.) $\vec{r}(t)=\langle t\cos t,t\sin t,t\rangle$

Problem 40 Let $\vec{\ell}(t)=(x_0,y_0,z_0)+t\langle a,b,c\rangle$ where a,b,c are constant and (x_0,y_0,z_0) is a given point. Calculate the arclength as a function of time t using $t=0$ as your base time. Does the value of your arclength function at $t=1$ make sense given the geometry in play for this curve ?