Same instructions as Mission 1. Thanks!

Problem 41 Find the parametrization of the tangent line to $\vec{r}(t) = \langle e^t,\ 3t+7,\ t^3\rangle$ at the point where this curve intersects the $x = 1$ plane.

Problem 42 Find the T, N, B as well as the curvature and torsion of

$$\vec{\gamma}(t) = \langle 2+3t,\ 2+4\cos t,\ 1+4\sin t\rangle.$$

Box answers for credit.

Problem 43 Let $\vec{r}(t) = \langle 3\cos 2t, 4, 3\sin 2t\rangle$ for $0 \leq t \leq 2\pi$. Find the T, N, B frame and calculate the curvature and torsion of this curve.

Problem 44 Suppose $\vec{\gamma}(t) = \langle 1+3\sin t, 2+4t, 3+3\cos t\rangle$ for $t \in \mathbb{R}$. Find:

(a.) the colliding tangent line at $(4, 2+2\pi, 3)$

(b.) the $\vec{T}, \vec{N}, \vec{B}$ frame

(c.) the curvature and torsion

Problem 45 Suppose $x = e^{-t}\cos(t)$ and $y = e^{-t}\sin(t)$ and $z = e^{-t}$ for $0 \leq t \leq 4\pi$. Calculate and simplify the tangent, normal and binormal vector fields for the curve parametrized by the given scalar parametric equations.

Problem 46 Calculate, and simplify as much as possible, the following derivative:

$$\frac{d}{dt}\left[\vec{r} \bullet (\vec{r}\,' \times \vec{r}\,'')\right].$$

Problem 47 Derive the following formula for curvature of a non-stop, non-linear path $\vec{r}$:

$$\kappa = \frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^3}.$$

Problem 48 Let $C(x) = \int_0^x \cos(t^2)dt$ and $S(x) = \int_0^x \sin(t^2)dt$ define the functions C and S known as **Fresnel integrals**. Let $\eta = \frac{1}{\sqrt[4]{2}}$. Define

$$\vec{\gamma}(t) = \langle \eta C(\eta t), \eta S(\eta t), \eta^2 t\rangle$$

for $t \geq 0$. Calculate the arclength, curvature and torsion for this curve at time t.

Problem 49 Consider a smooth path which allows the construction of the T,N,B frame.

(a.) If the path is planar then show the torsion $\tau = 0$,

(b.) If the torsion $\tau = 0$ then show geometrically the path lies in a plane.

Problem 50 Suppose two ninja begin travelling the paths given below. To begin, at $t = 0$, a relatively slow genin level ninja sets off in a NE direction given by

$$\vec{r}_1(t) = \langle -10 + t, 1 + t \rangle.$$

However, at the same time $t = 0$, an enemy Jonin sets off in a NW direction given by

$$\vec{r}_2(t) = \langle 20 - 4t, 6 + t \rangle.$$

Both of these paths are placed in a forest thick with a mist which lowers visability to near zero. Suppose the Jonin level ninja has advanced tracking skills that allow his to pick up on the faintest of scents. If he crosses the path of an enemy he can smell it and then alter his path to pursue and attack the enemy genin. Should the genin worry? Is he in danger? (a Jonin is no match for a typical genin in a usual battle, if the Jonin catches the genin it's game over for the lowly genin)

Problem 51 Suppose the velocity is given by $\vec{v}(t) = \langle t, 3, t\sinh(t^2) \rangle$ for some particle which has initial position $(1, 2, 3)$. Find for $t \geq 0$ the:

(a) acceleration at time t,

(b) position at time t,

(c) speed at time t,

(d) distance travelled at time t (in terms of an integral).

Problem 52 If the velocity of the Black and White Knight is given by $\vec{v}(t) = (1 + 2t\cos(t^2))\,\vec{v}_o$ where $\vec{v}_o$ is a constant vector. If the Black and White Knight is at the origin when $t = 0$ then find:

(a.) the acceleration $\vec{a}$ at time t,

(b.) the position $\vec{r}$ at time t.

Problem 53 Suppose $\vec{\gamma}(t) = \langle 3\cos(t^2), 3\sin(t^2), 2t + 1 \rangle$ is the position of a particle at time t. Find a_T and a_N components of the acceleration of this particle at time t.

Problem 54 Suppose $\vec{v} = 3t^2\vec{c}$ where $\vec{c}$ is a constant vector gives the velocity of a ninja. If the position of the ninja is $\vec{r}_o$ at time $t = 0$ then find:

(a.) find the acceleration of the ninja at time t,

(b.) the position of the ninja at time t,

(c.) the distance travelled from time zero to time t by the ninja.

Problem 55 Suppose $\vec{\gamma}$ is the path for which T, N, B are well-defined. It is given for this problem that curvature is given by $\kappa = \dfrac{\|\gamma' \times \gamma''\|}{\|\gamma'\|^3}$ and torsion by $\tau = \dfrac{(\gamma' \times \gamma'') \bullet \gamma'''}{\|\gamma' \times \gamma''\|^2}$. Let $\vec{B}, \vec{C}$ be constant orthogonal unit-vectors and suppose f and g are smooth functions of time t for which $f'g'' - g'f'' \neq 0$ for all t. If $\vec{r}_o$ is the initial position and

$$\vec{r}(t) = \vec{r}_o + f(t)\vec{B} + g(t)\vec{C}$$

is the position of a given ninja dog at time t. Then find the curvature and torsion of the dog's trajectory as functions of time t. Is the motion of this dog planar?

Problem 56 Let T be the triangle with vertices $(0,3)$, $(4,3)$ and $(2,0)$.

(a.) find the centroid of T,

(b.) If $\displaystyle\int_T (ax^2+y^2)ds = \frac{4396+2162\sqrt{13}}{3}$ then find the value of a (simplify answer).

Problem 57 Consider the helix C parametrized by $\vec{r}(t) = \langle 3\cos t, 4t, 3\sin t\rangle$ for $0 \leq t \leq 2\pi$:

(a.) find the centroid of C,

(b.) if $\lambda = \alpha x + \beta y$ where α, β are positive constants describes the linear mass density ($\lambda = \frac{dm}{ds}$) then find the center of mass of C.

Problem 58 Let $x = t^2\cos t$ and $y = t^2 \sin t$ parametrize a curve where we have $t \geq 0$. Find the arclength function for this curve where $s = 0$ for $t = 0$.

Problem 59 Let $C = C_R \cup D$ be the semi-circular path formed by joining the half-circle C_R from $(R,0)$ to $(-R,0)$ in the counter-clockwise fashion to the diameter D which goes from $(-R,0)$ to $(0,R)$. Let $\lambda(x,y) = x+y$ describe the density of radioactive weasels in the plane hence λds gives the amount of weasels along ds. Calculate the total number of weasels which a ninja encounters as he travels around C. If he hits more than 67 weasels then he dies. Does the ninja survive ? Answer in terms of R.

Problem 60 Consider the plane $\mathcal{P}_1$ parametrized by $\vec{r}(s,t) = (1,2,3) + s\langle 0,1,-1\rangle + t\langle 1,2,2\rangle$.

(a.) Find a parametrization for the circle which lies in the plane $\mathcal{P}_1$ with radius R and center $(1,2,3)$.

(b.) If the mass density $\lambda = 2z$ for the wire forming the circle then find its center of mass.