Same instructions as Mission 1. Thanks!

Problem 61 Let $S = [1,2] \times (3,4]$.

- **(a.)** Plot S and show why it is not an open set.
- **(b.)** Explain why S is not a closed set.
- **(c.)** Find ∂S and express it as a union of sets.
- **(d.)** Is $S_2 = S - \partial S$ an open or closed set ? Explain.
- **(e.)** Is $S_3 = S \cup \partial S$ an open or closed set ? Explain.

Problem 62 Let $A = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 4\}$ and $B = \{(x,y) \in \mathbb{R}^2 \mid x^2 + (y+2)^2 < 4\}$.

- **(a.)** Is $A \cap B$ an open set ?
- **(b.)** Is $A \cup B$ a path connected set ?
- **(c.)** Show $A \cup B \cup \partial A$ is a path connected set
(proof by picture and a few sentences will suffice)

Problem 63 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

Problem 64 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(2,-1)} \frac{(x-2)(y+1)}{(x-2)^2 + (y+1)^2}$$

Problem 65 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y^2}{x^2 + y^2}$$

Problem 66 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(-1,0)} \frac{\ln(2 + x^2 + 2x + y^2)}{x^2 + 2x + y^2 + 1}$$

Problem 67 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2} - 1}{x^2 + y^2}$$

Problem 68 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)^2}{x^2+y^2}$$

Problem 69 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(3,4)} \frac{(x-3)^2(y-4)^2}{(x^2-6x+y^2-8y+25)^{3/2}}$$

Problem 70 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}$$

Problem 71 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{(x^2+y^2)^2}$$

Problem 72 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y,z)\to(a,b,c)} \frac{xyz}{x^2+y^2+z^2},$$

(break into cases as needed)

Problem 73 Either show the limit exists by a change of coordinates if necessary or show the limit does not exist using an appropriate theorem.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^2-y^2}{x^2+y^2+z^2}$$

Problem 74 Let $f(x,y,z) = \begin{cases} A+\cos(xyz) & : (x,y,z)=(0,0,0) \\ \frac{\sin(x^2+y^2+z^2)}{x^2+y^2+z^2} & : (x,y,z)\neq(0,0,0) \end{cases}$. For what value of A is f continuous on $\mathbb{R}^3$? Justify your claim.

Problem 75 Let $f(x,y) = \sqrt{x^2+y^2}$. Use the definition of the directional derivative to determine whether

$$D_{\widehat{u}}f(0,0)$$

exists for $\widehat{u} = \frac{1}{\sqrt{2}}\langle 1,1\rangle$.

Problem 76 Use the definition of the directional derivative to find $D_{\widehat{u}}f(0,0)$ for an arbitrary unit vector $\widehat{u} = \langle u_1, u_2 \rangle$ given

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Problem 77 Use the definition of partial derivatives to show $\frac{\partial x}{\partial x} = 1$ whereas $\frac{\partial x}{\partial y} = 0$.

Problem 78 Find the directional derivative of $f(x,y) = x^2 + 3xy - y^2$ at $(1,2)$ in the direction of $\langle 3, 4 \rangle$.

Problem 79 Let $f(x,y) = xe^y$. Find the directional derivative at $P = (1,0)$ in the direction from P toward $Q = (4,4)$.

Problem 80 Let $f(x,y) = x^2y + 3y^2$ and define $P = (1,2)$:

(a.) Find the gradient $(\nabla f)(P)$.

(b.) Find the direction in which f increases most rapidly.

(c.) Find the maximum rate of increase.