Same instructions as Mission 1. Thanks!

Problem 81 Let $f(x, y) = x^2 + xy + y^2$ and $P = (1, 1)$.

(a.) If possible, find the direction(s) in which f changes at rate of 5 at P

(b.) If possible, find the direction(s) in which f changes at rate of 3 at P

Problem 82 Let $f(x, y, z) = x^2y + yz^2 - xz$. Find the directional derivative of f at $P = (1, -1, 2)$ in the direction of $\langle 2, -1, 2\rangle$.

Problem 83 Let $T(x, y, z) = 100e^{-(x^2+y^2+z^2)/4}$ describe the temperature at (x, y, z). Let $P = (1, 1, 0)$ and $Q = (3, 2, 1)$. Find the rate at which the temperature changes at P as one moves in the direction toward Q.

Problem 84 Let $f(x, y) = \dfrac{x}{x^2 + y^3}$

(a.) Calculate $\dfrac{\partial f}{\partial x}$

(b.) Calculate $\dfrac{\partial f}{\partial y}$

(c.) Calculate $\dfrac{\partial^2 f}{\partial x^2}$

(d.) Calculate $\dfrac{\partial^2 f}{\partial y^2}$

(e.) Calculate $\dfrac{\partial^2 f}{\partial x \partial y}$

Problem 85 A basic wave equation is

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}.$$

The wave can be viewed as a graph in the xy-plane which animates with time t. Calculate appropriate partial derivatives to test if the functions give a solution to the given wave equation. ($c_1, c_2, v \neq 0$ are constants)

(a.) $y(x, t) = x^3 - vt^3$

(b.) $y(x, t) = c_1 \sin(x - vt) + c_2 \cos(x - vt)$

(c.) $y(x, t) = \sin(vt)\sin(x)$

Problem 86 Let $f(x, y, z) = z^2 \ln(x + y^2)$. Calculate the following:

(a.) f_{xx}

(b.) f_{xy}

(c.) f_{yy}

(d.) f_{zz}

(e.) f_{xxxzzz}

Problem 87 For each function below, calculate ∇f

(a.) $f(x,t) = e^{-t}\cos(\pi x)$

(b.) $f(x,t) = \sqrt{x+2}\,\ln(t+3)$

(c.) $f(x,y) = \tan(xy)$

(d.) $f(x,y) = \dfrac{ax+by}{cx+dy}$ where a,b,c,d are constants

Problem 88 For each function below, calculate ∇f

(a.) $f(u,v) = \dfrac{e^v}{u+v^2}$

(b.) $f(u,v) = (u^2v - v^3)^5$

(c.) $f(r,\theta) = \sin(r\cos\theta)$

(d.) $f(x,y) = \ln(x + \sqrt{x^2+y^2})$

Problem 89 Consider $f(x,y) = 3x^2 - y$

(a.) plot the level curves $f(x,y) = k$ for $k = -2,-1,0,1,2$,

(b.) calculate ∇f,

(c.) find curves which are tangent to ∇f,

(d.) how do the curves in (a.) and those in (c.) relate ?

Problem 90 Consider $f(x,y) = x^2 - 2y^2$

(a.) plot the level curves $f(x,y) = k$ for $k = -2,-1,0,1,2$,

(b.) calculate ∇f,

(c.) sketch ∇f

(d.) what does ∇f tell you about the level curves of f at p provided $(\nabla f)(p) \neq 0$?

Problem 91 Calculate dw for each expression below:

(a.) $w = \ln(x+2y+3z)$

(b.) $w = ze^{xyz}$

(c.) $w = \frac{y}{x+y+z}$

(d.) $w = 3^{\alpha\beta^2\gamma^3}$

Problem 92 Let $z = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$. Calculate z_x, z_y, z_{xx}, z_{xy} and z_{yy}. Simplify your answers.

Problem 93 Let $z = \sin(xy^2)$. Furthermore, $x = \alpha + \beta^2$ and $y = \beta^2\sin(\beta)$. Use the multivariate chain rule to calculate $\dfrac{\partial z}{\partial \alpha}$ and $\dfrac{\partial z}{\partial \beta}$.

Problem 94 Suppose $h_x(1,2) = 33$ and $h_y(1,2) = 17$. In addition, $x(u,v) = uv^2$ and $y(u,v) = u^2 - v$. Let $g(u,v) = h(x(u,v),\, y(u,v))$ and calculate $\dfrac{\partial g}{\partial u}(1,-1)$.

Problem 95 You are stuck on the surface $xy^2z^3 = 1$. At the point $(1,1,1)$ you measure your x-velocity to be $2ft/s$ and your y-velocity is $-3ft/s$. Assuming units of ft and s find $\frac{dz}{dt}$ and determine your speed.

Problem 96 Let $\alpha = x + y$ and $\beta = 2x - y$. Derive Laplace's equation in α, β coordinates. That is, rewrite $u_{xx} + u_{yy} = 0$ in terms of derivatives with respect to α, β coordinates.

Problem 97 Given that $x = u^2 + v^2$ and $y = 3uv$ and $z = 3\sin(uv)$ and $w = ze^{xy}$ calculate w_u and w_v and finally w_z.

Problem 98 Calculate the Jacobian matrix of $\vec{r}(u,v) = \langle u^2+v^2,\ 3uv,\ 3\sin(uv)\rangle$ and that of $f(x,y,z) = ze^{xy}$. Multiply these matrices and identify how this relates to the previous problem.

Problem 99 Let $f(x,y) = \sqrt{9 + x^2y^2}$.

(a.) Find the linearization of $f(x,y)$ based at (a,b),

(b.) Find the equation of the tangent plane at $(a, b, f(a,b))$.

(c.) Suppose a ninja dog runs along $z = f(x,y)$ such that it has $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = -3$ at $(2,-2,5)$. Find the speed of the dog.

Problem 100 Let $f(x,y,z) = \sin(x)\cos(y) + \sin(\pi z)$.

(a.) Find the linearization of $f(x,y,z)$ based at $(\pi/2, \pi/3, 1)$.

(b.) Find the rate of change of f in the $\langle 1,1,1\rangle$-direction at $(\pi/2, \pi/3, 1)$.

(c.) Which directions does f change at the rate of 4 at $(\pi/2, \pi/3, 1)$.

(d.) Which directions does f change at the rate of 0 at $(\pi/2, \pi/3, 1)$.