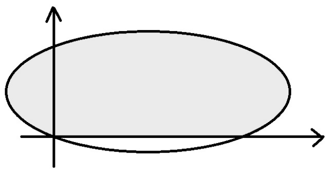
Same instructions as Mission 1. Thanks!

Problem 141 Calculate. Draw a picture to guide your logic.

$$\int_0^2 \int_{y^2}^4 y^3 e^{x^3}\, dx dy$$

Problem 142 A portion of the region bounded by $(x-2)^2 + 4(y-1)^2 = 8$ is graphed below. Denote the shaded region by R. Set-up the integral for $\iint_R f dA$.



Problem 143 Calculate $\displaystyle\int_0^1 \int_0^x \int_{x^2+y^2}^{x^3+y^3} 2z\, dz\, dy\, dx$

Problem 144 Reverse the order of integration in order to calculate the following integral:

$$\int_0^1 \int_y^1 \frac{2}{1+x^4}\, dx\, dy.$$

Problem 145 Let B be the solid region bounded by $x = 0$, $y = 0$, $z = 0$ and the plane $2x+2y-3z = 1$. Calculate the volume of B.

Problem 146 Let R be the regions in quadrant I bounded by $x^2+y^2 = 1$ and $x^2+y^2 = 4$ and $y = x$ and $y = 3x$. Calculate $\displaystyle\iint_R \sqrt{x^2+y^2}\, dA$ by changing the integration to polar coordinates.

Problem 147 Find the volume bounded by the cylinder $x^2 + y^2 = 1$ and $z = 2 + x + y$ and $z = 1$.

Problem 148 Let B be a ball of radius R centered at the origin. Calculate $\displaystyle\iiint_B e^{-\rho^3}\, dV$.

Problem 149 Find the centroid of the rightmost region bounded by $r = 4\cos\theta$ and $r = 2$.

Problem 150 Suppose the density of a solid sphere B of radius R is given by $\delta = \alpha\rho^c$ where α, c are constants and $\rho = \sqrt{x^2+y^2+z^2}$. The units of the constants are chosen as to give $\delta = \frac{dm}{dV}$ units of mass per volume. The moment of intertia with respect to the z axis is given by $I = \iiint_B r^2 \delta dV$ where $r^2 = x^2 + y^2$. Calculate I as a function of the constants c and α.

Problem 151 Calculate $\displaystyle\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left(12 - x^2 - y^2 - \sqrt{x^2+y^2}\right) dy\, dx$ by changing to an integral in polar coordinates.

Problem 152 Let P be the parallelogram with vertices $(0,0),(0,1),(2,4),(2,5)$. Calculate

$$\iint_P \sqrt{2x(y-2x)}\,dA$$

by making a substitution of $x = 2u$ and $y - 2x = v$. Notice, this substitution changes P to $[0,1]\times[0,1]$ in u, v space. Thanks to Brigg's and Cochran section 14.7 for this example.

Problem 153 Use the Jacobian to relate the volume element $du\,d\phi\,d\theta$ to the Cartesian volume element $dx\,dy\,dz$ given that

$$x = u\cos\theta\cosh\phi, \qquad y = u\sin\theta\cosh\phi, \qquad z = u\sinh\phi.$$

Problem 154 Suppose the mass per unit area is given by $\sigma(x,y) = x^2$. Find the center of mass for the rectangular region $R = [0,1]\times[0,1]$.

Problem 155 Calculate $\displaystyle\int_0^1\int_{2x}^2 e^{y^2}\,dy\,dx$

Problem 156 Suppose $R = \{(x,y) \mid 0 \leq x^2+y^2 \leq 9, \text{ and } x \geq 0\}$. Calculate $\displaystyle\iint_R \sin(x^2+y^2)\,dA$

Problem 157 Let B be part the ball of radius R centered at the origin $(x^2+y^2+z^2 \leq R^2)$ such that $x \geq 0$ and $y \geq 0$ and $z \geq 0$. Suppose $\delta(x,y,z) = \dfrac{1}{1+x^2+y^2+z^2}$ is the mass-density $\frac{dm}{dV}$ of B. Calculate the mass of B :

Problem 158 Calculate the volume of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 \leq 1$ where $a,b,c > 0$.

Problem 159 Calculate $\displaystyle\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} \left(zx^2+zy^2+z^3\right)\,dz\,dy\,dx.$

Problem 160 Let R be the diamond with vertices $(0,0),(1,1),(0,2)$ and $(-1,1)$. Change coordinates to calculate the integral:
note: $\int \sinh(u)\,du = \cosh(u)+c.$

$$\iint_R \sinh\left(\frac{x+y}{2}\right)\sin\left(\frac{y-x}{2}\right)dA$$